The Standard Nuclear (Shell) Model
A Primer

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OUTLINE

- Introduction
- Basics on the ISM (Interacting Shell Model)
- The physics of the neutron rich nuclei around N=20, 28 and 40
- Deformation and Superdeformation at N=Z
- Triaxiality; The light Xenon isotopes
- Quadrupole Collectivity; SU3 and its variants
- Glossary

The Nuclear A-body Problem

- In the Standard Nuclear Model the elementary components are nucleons (neutrons N and protons Z, N+Z=A). The mesonic and quark degrees of freedom are integrated out
- In most cases non-relativistic kinematics is used
- The bare nucleon-nucleon (or nucleon-nucleon-nucleon) interactions are inspired by meson exchange theories or more recently by chiral perturbation theory, and must reproduce the nn phase shifts, and the properties of the deuteron and other few body systems

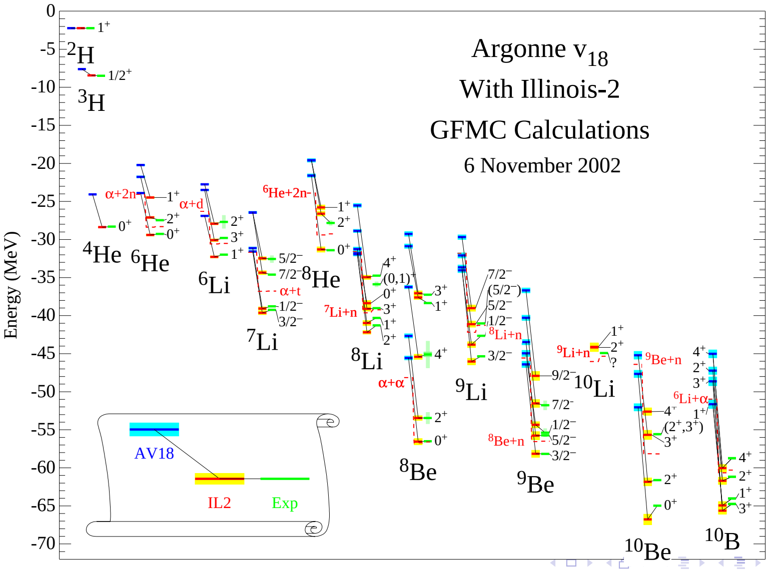
The Nuclear A-body Problem

- The challenge is to find $\Psi(\vec{r}_1, \vec{r}_2, \vec{r}_3, \ldots \vec{r}_A)$ such that
- HΨ=EΨ, with
- H=$\sum_{i}^{A} T_i + \sum_{i,j}^{A} V_{2b}(\vec{r}_i, \vec{r}_j) + \sum_{i,j,k}^{A} V_{3b}(\vec{r}_i, \vec{r}_j, \vec{r}_k)$
- The knowledge of the eigenvectors Ψ and the eigenvalues E make it possible to obtain electromagnetic moments, transition rates, weak decays, cross sections, spectroscopic factors, etc.

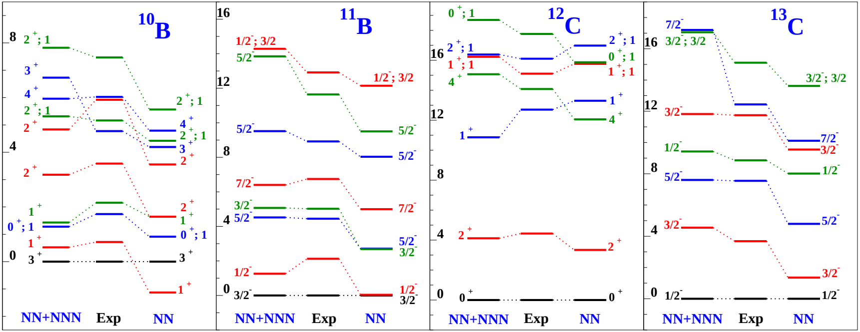
The Nuclear A-body Problem

- The task is indeed formidable
- Only very recently and only for very light nuclei A$\leq$10 the problem has been solved ”exactly”,
- thanks to the pioneer work of Pandharipande, Wiringa and Pieper, which used variational methods (Green Function) solved by Monte Carlo techniques, GFMC
- More recently, the perturbative approach has been implemented in the framework of the No Core Shell Model (NCSM) by Barrett, Navratil, and Vary

"Ab Initio" Approaches; GFMC



”Ab Initio” Approaches; NCSM



”Ab Initio” Approaches

- A very important outcome of these calculations is that it is compulsory to include three body forces in order to get correct solutions of the nuclear many body problem
- The GFMC and the NCSM are severely limited by the huge size of the calculations when A becomes larger than twelve
- For the rest of the chart of nuclides, approximate methods have to be used. Except for the semiclasical ones (liquid drop) and the α-cluster models, all are based on the Independent Particle Approximation
- Mid-way between the ”ab initio” and the conventional Shell Model and Energy Density Functional ones are new (or renewed) methods like the Coupled Cluster Expansion, the In-medium Similarity Renormalization Group and other Many Body Perturbation Methods

The Independent Particle Model

The basic idea of the IPM is to assume that, at zeroth order, the result of the complicated two body interactions among the nucleons is to produce an average self-binding potential. Mayer and Jensen (1949) proposed an spherical mean field consisting in an isotropic harmonic oscillator plus a strongly attractive spin-orbit potential and an orbit-orbit term. Later, other functional forms were adopted, *e.g.* the Woods-Saxon well

The usual procedure to generate a mean field in a system of N interacting fermions, starting from their free interaction, is the Hartree-Fock approximation, extremely successful in atomic physics. Whatever the origin of the mean field, the eigenstates of the N-body problem are Slater determinants *i.e.* anti-symmetrized products of N single particle wave functions.

The Independent Particle Model

In the nucleus, there is a catch, because the very strong short range repulsion and the tensor force make the HF approximation based upon the bare nucleon-nucleon force impracticable.

However, at low energy, the nucleus do manifest itself as a system of independent particles in many cases, and when it does not, it is due to the medium range correlations that produce strong configuration mixing and not to the short range repulsion.

The Independent Particle Model

Does the success of the shell model really "prove" that nucleons move independently in a fully occupied Fermi sea as assumed in HF approaches? In fact, the single particle motion can persist at energies in fermion systems due to the suppression of collisions by Pauli exclusion (Pandharipande et al., RMP69)

Brueckner theory takes advantage of the Pauli blocking to regularize the bare nucleon- nucleon interaction, in the form of density dependent effective interactions of use in HF calculations or G-matrices for large scale shell model calculations.

The Independent Particle Model

The wave function of the ground state of a nucleus in the IPM is the product of an Slater determinant for the Z protons that occupy the Z lowest states in the mean field and another Slater determinant for the N neutrons in the N lowest states of the mean field

In second quantization, this state can be written as:

$$|N\rangle \cdot |Z\rangle$$

with

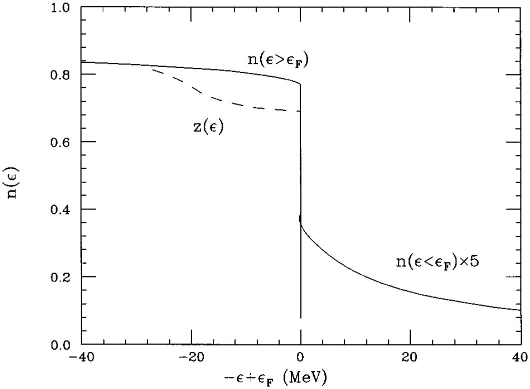
$$|N\rangle = n_1^\dagger n_2^\dagger \dots n_N^\dagger |0\rangle$$

$$|Z\rangle = z_1^\dagger z_2^\dagger \dots z_Z^\dagger |0\rangle$$

It is obvious that the occupied states have occupation number 1 and the empty ones occupation number 0

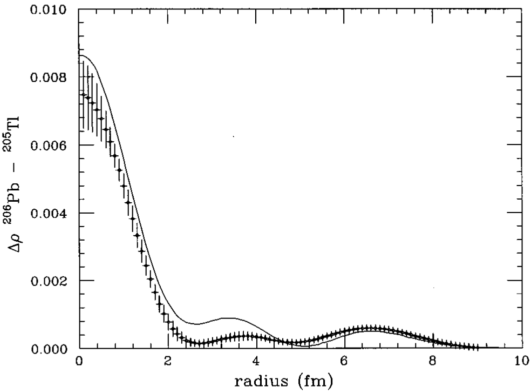
The Meaning of the Shell Model

Dilution of the Spectroscopic strength by the bare N-N interaction. Results for nuclear matter.



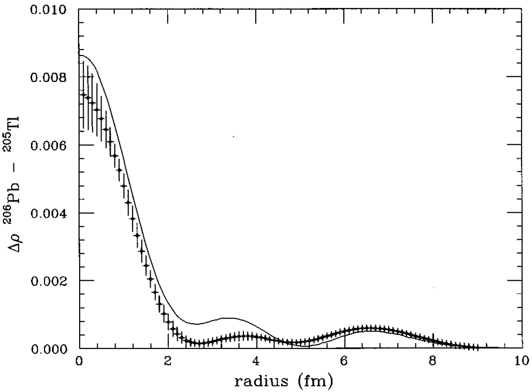
If we had a system of non interacting fermions, the figure would be a step function with occupation 1 below the Fermi level and 0 above

The Meaning of the Shell Model



In spite of that, the nuclear quasi-particles resemble extraordinarily to the mean field solutions of the IPM, as can be seen in the classical example of the charge density difference between ^{206}Pb and ^{205}Tl, fromthe electron scattering experiments of Cavedon *et al, 1982*

The Meaning of the Shell Model



The shape of the $3s_{1/2}$ orbit is very well given by the mean field calculation. To make the agreement quantitative the calculated density has to be scaled down by the occupation number

To know more, Read the article " Independent particle motion and correlations in fermion systems" V. R. Pandharipande, et al., RMP 69 (1997) 981.

Vocabulary

- state: a solution of the Schroedinger equation with a one body potential; e.g. the H.O. or the W.S. It is characterized by the quantum numbers *nljm* and the projection of the isospin t_z
- orbit: the ensemble of states with the same *nlj*, e.g. the $0d_{5/2}$ orbit
- shell: an ensemble of orbits quasi-degenerated in energy, e.g. the *pf* shell
- magic numbers: the numbers of protons or neutrons that fill orderly a certain number of shells
- gap: the energy difference between two shells
- SPE, single particle energies, the eigenvalues of the IPM hamiltonian
- ESPE, effective single particle energies, the eigenvalues of the monopole hamiltonian

The Interacting Shell Model (ISM)

Is an approximation to the exact solution of the nuclear A-body problem using effective interactions in restricted spaces

The effective interactions are obtained from the bare nucleon-nucleon interaction by means of a regularization procedure aimed to soften the short range repulsion. In other words, using effective interactions we can treat the A-nucleon system in a basis of independent quasi-particles

A Shell Model calculation amounts to diagonalizing the nuclear hamiltonian in the basis of all the Slater determinants that can be built distributing the valence particles in a set of orbits which is called valence space. The orbits that are always full form the core.

The three pillars of the shell model

- The Effective Interaction
- Valence Spaces
- Algorithms and Codes

E. Caurier, G. Martínez-Pinedo, F. Nowacki, A. Poves and A. P. Zuker. "The Shell Model as a Unified View of Nuclear Structure", RMP 77 (2005) 427.

Making the Effective Interaction Simple

The effective shell model interactions appears sometimes as a long list of meanigless numbers; the two body matrix elements of the Hamiltonian.

Without loosing the simplicity of the Fock space representation, we can recast these numbers in a way full of physical insight, following Dufour-Zuker rules

Any effective interaction can be split in two parts:
$\mathcal{H} = \mathcal{H}_m$(monopole) + $\mathcal{H}_M$(multipole).
$\mathcal{H}_m$ contains all the terms that are affected by a spherical Hartree-Fock variation, hence it is responsible for the global saturation properties and for the evolution of the spherical single particle energies

The Monopole Hamiltonian

$$\mathcal{H}_m = H_{sp} + \sum \left[\frac{1}{(1+\delta_{ij})} a_{ij}\, n_i (n_j - \delta_{ij}) \right.$$

$$\left. + \frac{1}{2} b_{ij} \left(T_i \cdot T_j - \frac{3n_i}{4} \delta_{ij} \right) \right].$$

The coefficients *a* and *b* are defined in terms of the centroïds:

$$V_{ij}^{T} = \frac{\sum_J V_{ijij}^{JT} [J]}{\sum_J [J]}$$

as: $a_{ij} = \frac{1}{4}(3V_{ij}^1 + V_{ij}^0)$, $b_{ij} = V_{ij}^1 - V_{ij}^0$, the sums run over Pauli allowed values.

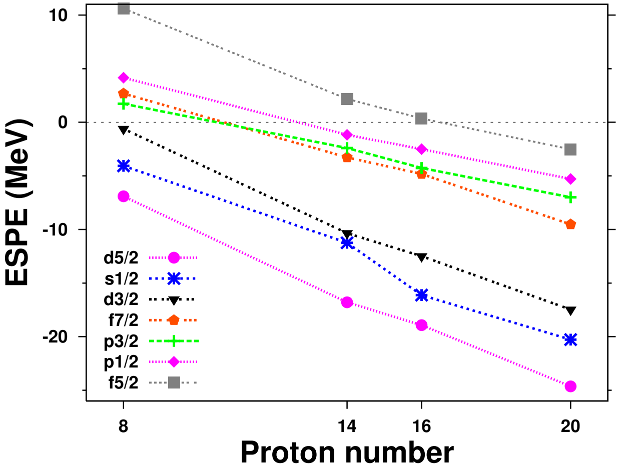
The Monopole Hamiltonian

The evolution of effective spherical single particle energies with the number of particles in the valence space is dictated by $\mathcal{H}_m$. In the case of identical particles the expression reads:

$$\epsilon_j(n) = \epsilon_j(n=1) + \sum_i V_{ij}^1 n_i$$

Even small defects in the centroids can produce large changes in the relative position of the different configurations due to the appearance of quadratic terms involving the number of particles in the different orbits.

The Drift of the Single Particle Energies



The Drift of the Single Particle Energies

Sometimes, the two body interaction is analyzed in terms of its central, spin-orbit and tensor components. Indeed, only the monopole parts of these terms can contribute to the evolution of the single particle energies. Nowacki and Sieja have shown recently that the main drivers of this evolution can be of any of the three types depending on which orbits are involved. Holt, Otsuka and Schweck have also shown that the monopole component of the three body force is responsible for the shell structure around ^{28}O.

So please refrain from putting always the blame on the tensor force. It is only in very few cases that it happens to be so

The Multipole Hamiltonian

$\mathcal{H}_M$ can be written in two representations, particle-particle and particle-hole:

$$\mathcal{H}_M = \sum_{r\leq s, t\leq u, \Gamma} W^{\Gamma}_{rstu} Z^{+}_{rs\Gamma} \cdot Z_{tu\Gamma},$$

$$\mathcal{H}_M = \sum_{rstu\Gamma} [\gamma]^{1/2} \frac{(1+\delta_{rs})^{1/2}(1+\delta_{tu})^{1/2}}{4} \omega^{\gamma}_{rtsu} (S^{\gamma}_{rt} S^{\gamma}_{su})^0,$$

where Z^{+}_{Γ} (Z_{Γ}) is the coupled product of two creation (annihilation) operators and S^{γ} is the coupled product of one creation and one annihilation operator.

$$Z^{+}_{rs\Gamma} = [a^{\dagger}_r a^{\dagger}_s]^{\Gamma} \text{ and } S^{\gamma}_{rs} = [a^{\dagger}_r a_s]^{\gamma}$$

The Multipole Hamiltonian

The W and ω matrix elements are related by a Racah transformation:

$$\omega_{rtsu}^{\gamma} = \sum_{\Gamma} (-)^{s+t-\gamma-\Gamma} \left\{ \begin{array}{ccc} r & s & \Gamma \\ u & t & \gamma \end{array} \right\} W_{rstu}^{\Gamma} [\Gamma],$$

$$W_{rstu}^{\Gamma} = \sum_{\gamma} (-)^{s+t-\gamma-\Gamma} \left\{ \begin{array}{ccc} r & s & \Gamma \\ u & t & \gamma \end{array} \right\} \omega_{rtsu}^{\gamma} [\gamma].$$

The operators $S_{rr}^{\gamma=0}$ are just the number operators for orbits r and $S_{rr'}^{\gamma=0}$ the spherical HF particle hole vertices. Both must have null coefficients if the monopole hamiltonian satisfies HF self-consistency.

The Multipole Hamiltonian

The operator $Z^{+}_{rr\Gamma=0}$ creates a pair of particle coupled to J=0 (or coupled to L=0 and S=0, or in a state of zero total momentum). Therefore the terms

$$Z^{+}_{rr\Gamma=0} \cdot Z_{ss\Gamma=0}$$

represent different pairing hamiltonians, whose specificities determine the values of the matrix elements $W^{\Gamma=0}_{rrss}$

The Multipole Hamiltonian

The operators S_{rs}^{γ} are typical vertices of multipolarity γ. For instance, γ=(J=1,L=0,T=1) produces a $(\vec{\sigma} \cdot \vec{\sigma})$ $(\vec{\tau} \cdot \vec{\tau})$ term which is nothing but the Gamow-Teller component of the nuclear interaction

The terms S_{rs}^{γ} γ=(J=2,T=0), that appear in interaction are of quadrupole type $r^2 Y_2$. They are responsible for the existence of deformed nuclei, and are specially large and attractive when $j_r - j_s$=2 and $l_r - l_s$=2.

Universality of the Multipole Hamiltonian

Indeed, a careful analysis of the effective nucleon-nucleon interaction in the nucleus, reveals that the multipole hamiltonian is universal and dominated by BCS-like isovector and isoscalar pairing plus quadrupole-quadrupole and octupole-octupole terms of very simple nature ($r^\lambda Y_\lambda \cdot r^\lambda Y_\lambda$)

Interaction	particle-particle		particle-hole		
	JT=01	JT=10	$\lambda\tau$=20	$\lambda\tau$=40	$\lambda\tau$=11
KB3	-4.75	-4.46	-2.79	-1.39	+2.46
FPD6	-5.06	-5.08	-3.11	-1.67	+3.17
GOGNY	-4.07	-5.74	-3.23	-1.77	+2.46
GXPF1	-4.18	-5.07	-2.92	-1.39	+2.47
BONNC	-4.20	-5.60	-3.33	-1.29	+2.70

The Effective Interaction

The evolution of the spherical mean field in the valence spaces remains a key issue, because we know since long that something is missing in the monopole hamiltonian derived from the realistic NN interactions, be it through a G-matrix, V_{low-k} or other options.

The need for three body forces is now confirmed. Would they be reducible to simple monopole forms? Would they solve the monopole puzzle of the ISM calculations? The preliminary results seem to point in this direction

The multipole hamiltonian does not seem to demand major changes with respect to the one derived from the realistic nucleon-nucleon potentials

The Valence Space(s)

An ideal valence space should incorporate the most relevant degrees of freedom AND be computationally tractable

Classical $0\hbar\omega$ valence spaces are provided by the major oscillator shells *p*, *sd* and *pf* shells

Other physically sound and computationally accessible valence spaces are proposed below.

(note: in a major HO shell of principal quantum number **p** the orbit j=p+1/2 is called *intruder* and the remaining ones are denoted by r_p)

Valence Space(s)

Miscellanea of computationally accessible (and physically sound) valence spaces:

- For very neutron rich nuclei around N=28, a good choice is to take the *sd* shell for protons and the *pf* shell for neutrons.
- r_2-*pf*: intruders around N and/or Z=20
- r_3-$g9/2(d5/2)$: ^{76}Ge, ^{82}Se, and the region around ^{80}Zr
- r_3-$g9/2(d5/2)$ for the neutrons and *pf* for protons: neutron rich Cr, Fe, Ni, Zn
- r_4-$h_{11/2}$ for neutrons and $p_{1/2} - g_{9/2}$-r_4 for protons: ^{96}Zr, ^{100}Mo,^{110}Pd, ^{116}Cd
- r_4-$h_{11/2}$ for neutrons and protons: ^{124}Sn, $^{128-130}$Te, ^{136}Xe

Algorithms and Codes

Algorithms include Direct Diagonalisation, Lanczos, Monte Carlo Shell Model, Quantum Monte Carlo Diagonalization, DMRG etc. There are also a number of different extrapolation ansatzs

The Strasbourg-Madrid codes (Antoine, Nathan), can deal with problems involving basis of 10^{10} Slater determinants, using relatively modest computational resources. Other competitive ones in the market are OXBACH, NUSHELL and MSHELL

Collectivity in Nuclei

The widespread presence of nuclei with deformed shapes is a conspicuous manifestation of the importance of the quadrupole-quadrupole terms in the nuclear multipole hamiltonian. Nuclear superfluidity (and the shift of the mass parabolas in even isobaric multiplets, and many other effects) signal also the importance of the pairing terms.

For a given interaction, a many body system would or would not display coherent features at low energy depending on the structure of the mean field around the Fermi level.

Nuclear Needles and Superfluids

If the spherical mean field around the Fermi surface makes the the pairing interaction dominant, the nucleus becomes superfluid

If the quadrupole-quadrupole interaction is dominant the nucleus acquires permanent deformation

In the extreme limit in which the monopole hamiltonian is negligible, the multipole interaction would produce superfluid nuclear needles.

Magic nuclei are spherical despite the strong multipole interaction, because the large gaps in the nuclear mean field at the Fermi surface block the correlations

The Symmetries of the Quadrupole Interaction

The isotropic harmonic oscillator has SU(3) symmetry. The quadrupole operators are generators of this group and the Casimir of the group contains the quadrupole-quadrupole interaction. Therefore the states of lower energy are those with maximal deformation compatible with the Pauli principle.

The spin orbit interaction breaks the SU(3) symmetry, but other SU3 variants emerge when there are favorable orbits around the Fermi level, like Pseudo-SU3 or Quasi-SU3.

The most popular “flaws” of the standard SM description

- Quadrupole effective charges are needed (But their value is universal and rather well understood)
- Spin operators are quenched by another universal factor which relates to the regularization of the interaction (also known as short range correlation). Indeed, BMF approaches share this shortcoming
- Not all the regions of the nuclear chart are amenable to a SM description yet

The mean field . . . and beyond

- HF-based approaches rely on the use of density dependent interactions of different sort; Skyrme, Gogny, RMF parametrizations
- The correlations are taken into account via symmetry breaking in the mean field
- Projections before (VAP) or after (PAV) variation are enforced to restore the conserved quantum numbers
- Ideally, configuration mixing is also implemented through the GCM

Physics Goals

The precision Spectroscopy towards larger masses

The description of the nuclear correlations in the laboratory frame

The change of the Magic Numbers far from Stability: The competing roles of spherical mean field and correlations

The precision descriptions of the weak nuclear processes. In particular the double β decay, which has the key to the nature of the neutrinos, the absolute scale of their masses and their hierarchy

The Nuclear Structure inputs for Nuclear Astrophysics