The Standard Nuclear (Shell) Model
A Primer

ALFREDO POVES
Departamento de Física Teórica and IFT, UAM-CSIC

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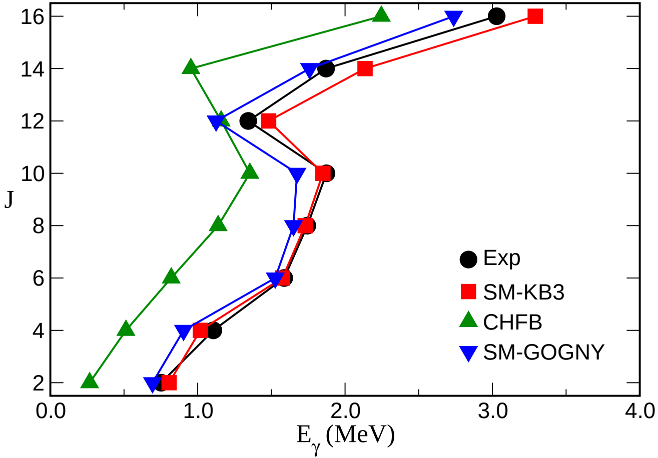
Part 3

- Deformation and Superdeformation at N=Z
- Triaxiality; The light Xenon isotopes
- Quadrupole Collectivity; SU3 and its variants
- Glossary

Collectivity at N=Z

Four protons and four neutrons on top of doubly magic ^{40}Ca, suffice to produce a well behaved rotor. In N=Z nuclei, a proper treatment of the neutron proton pairing, isovector and isoscalar is compulsory in order to reproduce the experimental moment of inertia.

ISM dialogues with BFM: the ^{48}Cr case



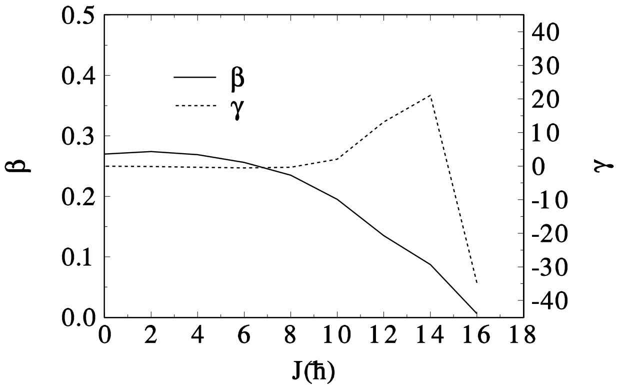
ISM dialogues with BMF: the ^{48}Cr case

Intrinsic states in the laboratory frame wave-functions.

J	$B(E2)_{exp}$	$B(E2)_{th}$	$Q_0(B(E2))$
2	321(41)	228	107
4	330(100)	312	105
6	300(80)	311	100
8	220(60)	285	93
10	185(40)	201	77
12	170(25)	146	65
14	100(16)	115	55
16	37(6)	60	40

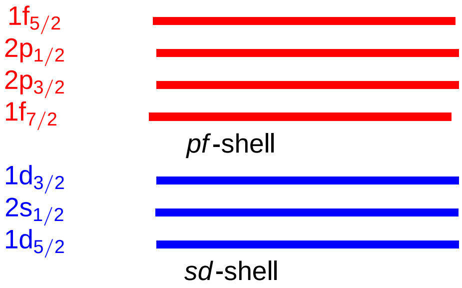
ISM dialogues with BMF: the ^{48}Cr case

The intrinsic state in the intrinsic description:



Coexistence: Spherical, Deformed and Superdeformed states in ^{40}Ca

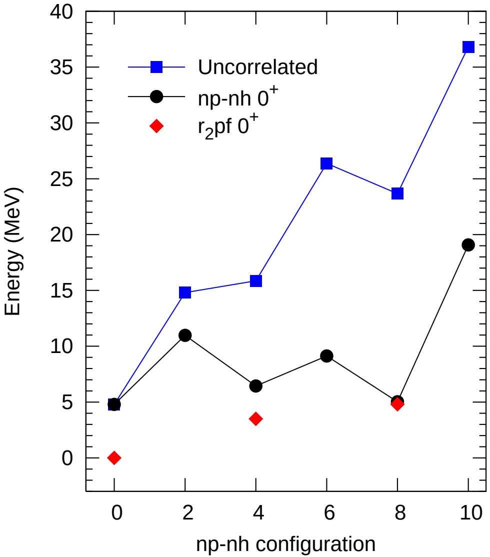
In the valence space of two major shells



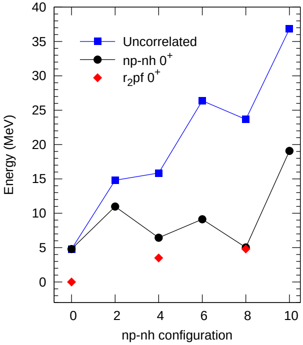
The relevant configurations are:

$[sd]^{24}$ 0p-0h in ^{40}Ca, SPHERICAL
$[sd]^{20}$ $[pf]^{4}$ 4p-4h in ^{40}Ca, DEFORMED
$[sd]^{16}$ $[pf]^{8}$ 8p-8h in ^{40}Ca, SUPERDEFORMED

The correlation energies

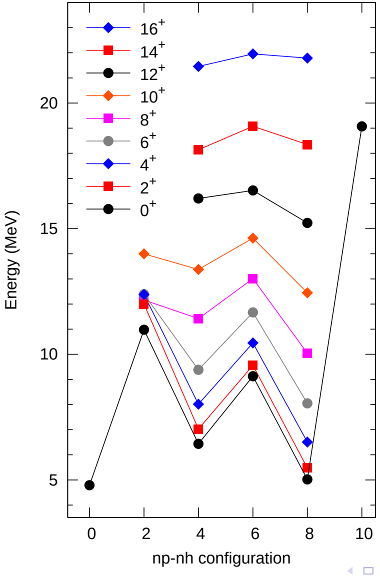


The correlation energies

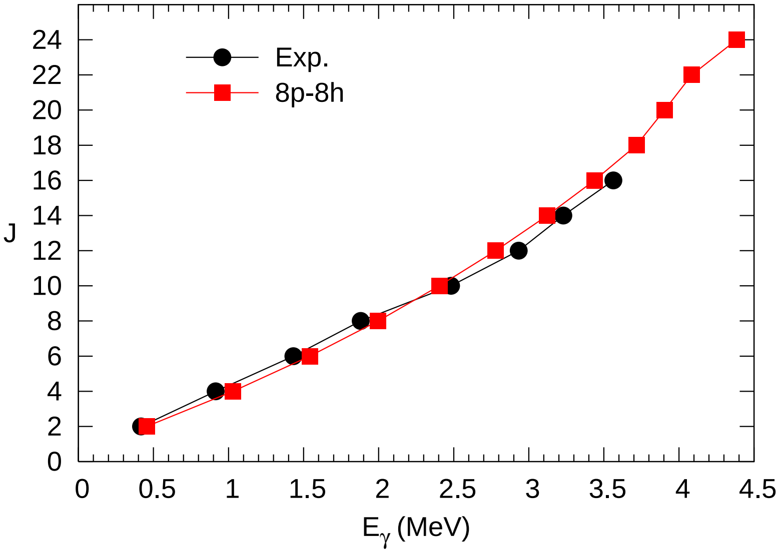


In the 8p-8h configuration the correlations amount to 18.5 MeV. 5.5 MeV are due to T=1 pairing and 0.5 MeV to T=0 pairing, thus the neutron-proton pairing contribution is 2.33 MeV. The remaining 12.5 MeV are most likely of quadrupole origin. In the 4p-4h configuration, the pairing contributions are the same, but the quadrupole is just 3.5 MeV. The physical gound state gains 5 MeV of pairing energy by mixing with the other np-nh states,

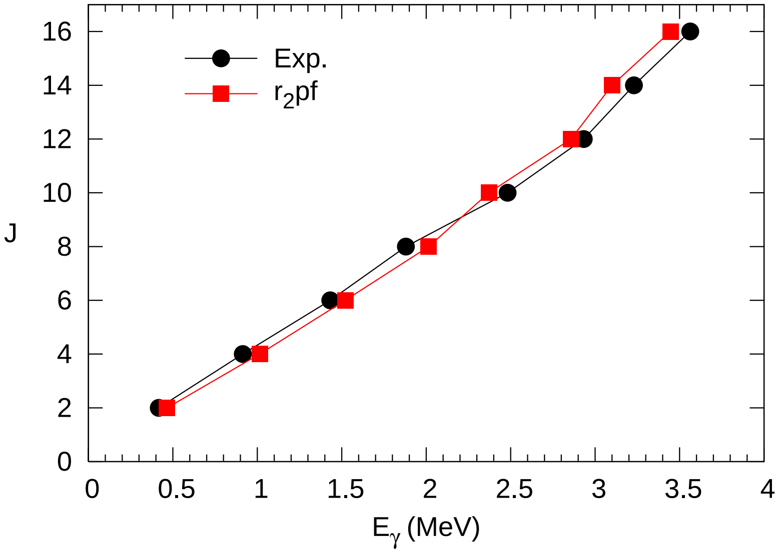
The np-nh energies as a function of J



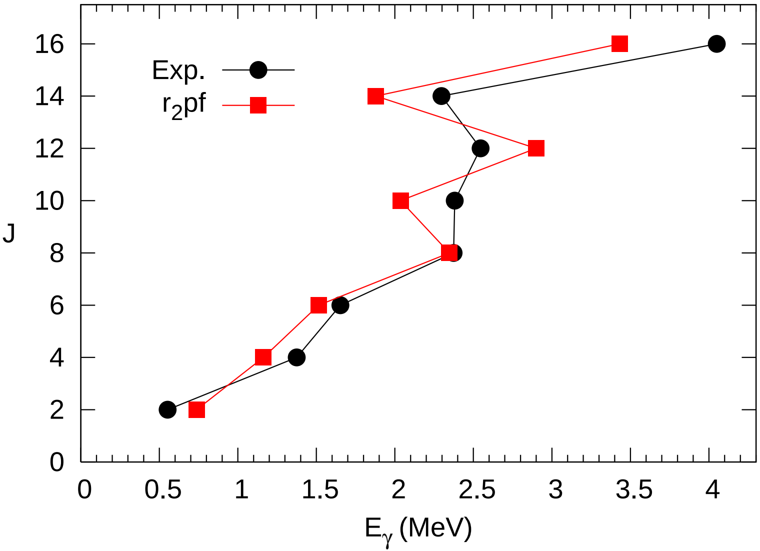
The superdeformed band: 8p-8h



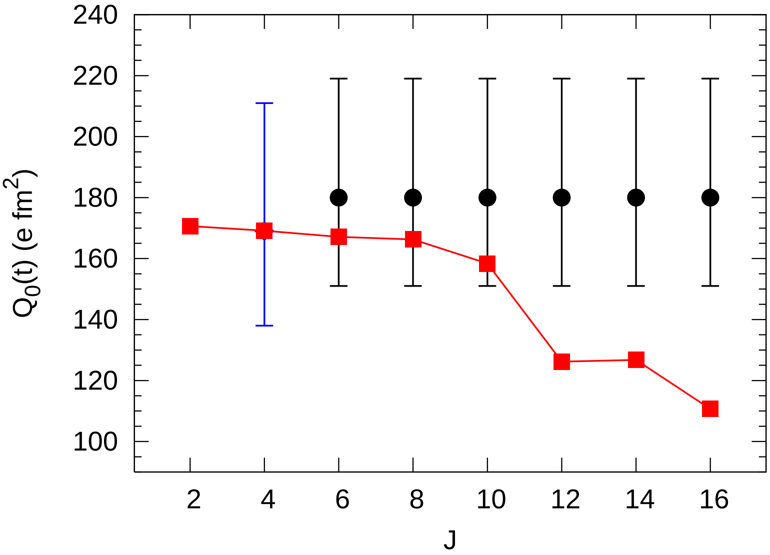
The Superdeformed band: Mixed calculation



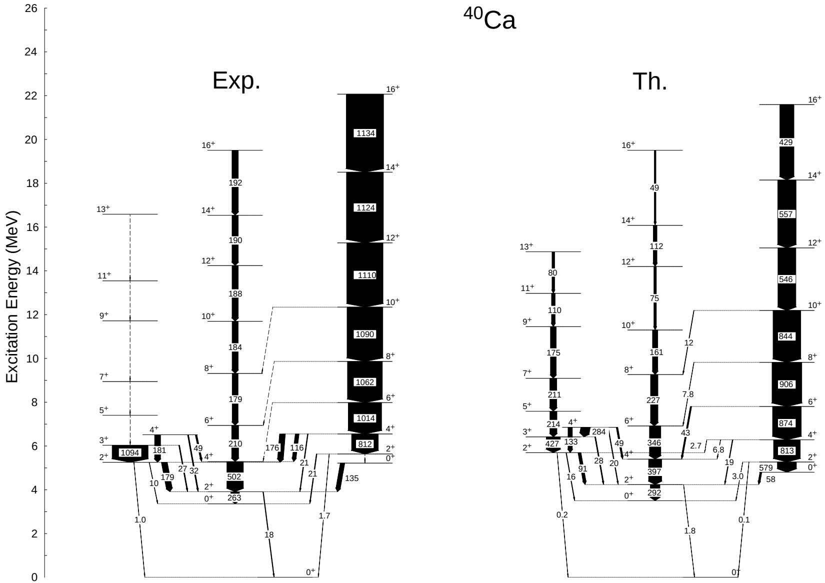
The triaxial deformed band: Mixed calculation



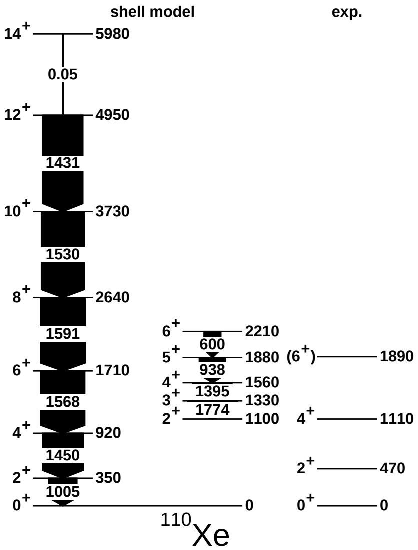
Transition Quadrupole Moments



Comparing with experiment



Collectivity in the light Xenon isotopes



Collectivity in light Xenon isotopes

J	E*	E_γ	BE2	Q_{sp}	Q_0 (BE2)	Q_0 (Q_{sp})	β
2^+	0.35	0.35	1005	-62	225	217	0.16
4^+	0.92	0.57	1450	-78	226	215	0.16
6^+	1.71	0.79	1568	-83	224	208	0.16
8^+	2.64	0.94	1591	-87	220	207	0.16
10^+	3.73	1.09	1530	-86	213	198	0.15
12^+	4.95	1.22	1431	-85	204	191	0.15
14^+	5.98	0.99	0.05	-126	1	279	
16^+	6.63	0.69	111	-125	56	273	
18^+	7.51	0.88	1184	-130	183	282	
20^+	8.51	1.00	1043	-134	172	288	

Collectivity in light Xenon isotopes

J	E*	E_γ	BE2	Q_{sp}	Q_0 (BE2)	Q_0 (Q_{sp})	β
2^+_2	1.10			+61			
3^+	1.33	0.23	1774	-1.3			
4^+_2	1.56	0.23	1395	-38	219	261	0.18
5^+	1.88	0.32	938	-54	217	234	0.17
6^+_2	2.21	0.33	600	-74	209	259	0.17

From the ratio $\frac{BE2(2^+_\gamma \rightarrow 2^+_y)}{BE2(2^+_\gamma \rightarrow 0^+_y)}$ $\gamma=20^o$.

$Q(2^+_\gamma)$=-$Q(2^+_y)$ and $Q(3^+)\approx 0$
suggest that the γ band can be labeled by K=2.

Collectivity in light Xenon isotopes

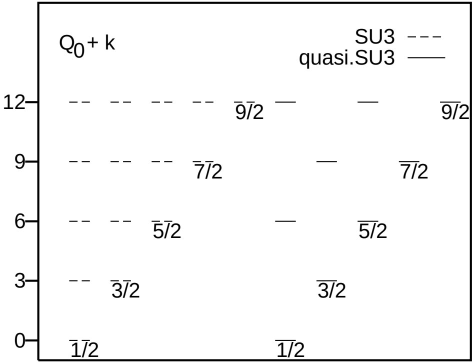
The valence space r4h contains a pseudo-SU3 triplet plus the intruder orbit $0h_{11/2}$. What happens if we remove the intruder orbit from the space?

- The moments of inertia of the bands get reduced by 30%
- The backbending is suppressed
- The triaxiality is reduced to $\gamma = 12^o$
- The magnetic moments are fully consistent with the rotational model up to J=20.

The mechanism of deformation and superdeformation in the laboratory frame

Consider the quadrupole force alone, taken to act in the p-th oscillator shell. It will tend to maximize the quadrupole moment, which means filling the lowest orbits obtained by diagonalizing the operator $Q_0 = 2q_{20} = 2z^2 - x^2 - y^2$. Using the cartesian representation, $2q_{20} = 2n_z - n_x - n_y$, we find eigenvalues $2p$, $2p - 3, \ldots$, etc. By filling the orbits orderly we obtain the intrinsic states for the lowest SU(3) representations: $(\lambda, 0)$ if all states are occupied up to a given level and (λ, μ) otherwise. For instance: putting two neutrons and two protons in the $K = 1/2$ level leads to the (4p,0) representation. For four neutrons and four protons, the filling is not complete and we have the (triaxial) (8($p - 1$),4) representation for which we expect a low lying γ band.

SU3 and Quasi-SU3



Nilsson orbits for SU(3) ($k = 2p$) and quasi-SU(3) ($k = 2p - 1/2$)

SU3 and Quasi-SU3

In *jj* coupling the angular part of the quadrupole operator $q^{20} = r^2 C^{20}$ has matrix elements

$$\langle j\, m | C^2 | j+2\, m \rangle \approx \frac{3[(j+3/2)^2 - m^2]}{2(2j+3)^2},$$

$$\langle j\, m | C^2 | j+1\, m \rangle = -\frac{3m[(j+1)^2 - m^2]^{1/2}}{2j(2j+2)(2j+4)}$$

The $\Delta j = 2$ numbers are—within the approximation made—identical to those in *LS* scheme, obtained by replacing j by l. The $\Delta j = 1$ matrix elements are small, both for large and small m, corresponding to the lowest oblate and prolate deformed orbits respectively. If the spherical j-orbits are degenerate, the $\Delta j = 1$ couplings, though small, will mix strongly the two $\Delta j = 2$ sequences (e.g., $(f_{7/2} p_{3/2})$ and $(f_{5/2} p_{1/2})$). The spin-orbit splittings will break the degeneracies and favour the decoupling of the two sequences. Hence the idea of neglecting the $\Delta j = 1$ matrix elements and exploit the correspondence

SU3 and Quasi-SU3

The resulting "quasi SU(3)" quadrupole operator respects SU(3) relationships, except for $m = 0$, where the correspondence breaks down. The resulting spectrum for quasi-$2q_{20}$ is shown together with the SU3 one. The result is not exact for the $K = 1/2$ orbits but a very good approximation. To check the validity of the decoupling, a Hartree calculation can be done for $H = \varepsilon H_{sp} + H_q$, where H_{sp} is the observed single particle spectrum in ^{41}Ca (essentially equidistant orbits with 2MeV spacings) and H_q is the quadrupole force in with a properly renormalized coupling. The result is exactly a Nilsson calculation,

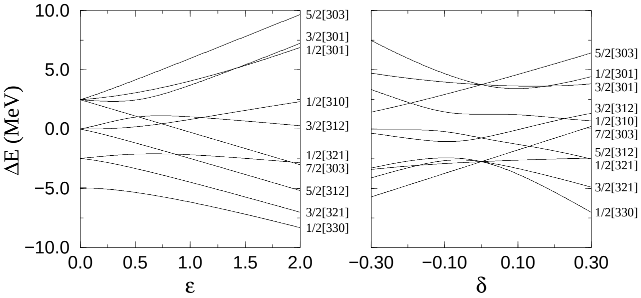
$$H_{mq0} = \hbar\omega \left(\varepsilon H_{sp} - \frac{\delta}{3} 2q_{20} \right),$$

where

$$\frac{\delta}{3} = \frac{1}{4} \frac{\langle 2q_{20} \rangle}{\langle r^2 \rangle} = \frac{\langle 2q_{20} \rangle}{(p + 3/2)^4}.$$

Quasi-SU3

Nilsson diagrams in the *pf* shell. Energy vs. single particle splitting ε (left panel), energy vs. deformation δ (right panel)



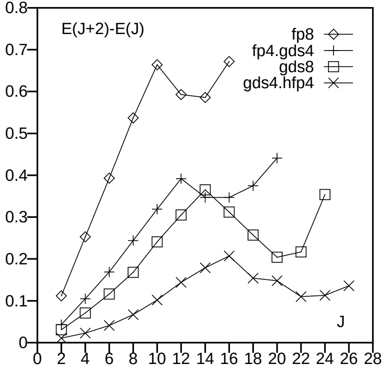
In the right panel of the figure the results are given in the usual form. In the left panel we have turned the representation around: since we are interested in rotors, we start from perfect ones (SU(3)) and let ε increase. At a value of ≈ 0.8 the four lowest orbits are in the same sequence as the right side of the

Quasi-SU3

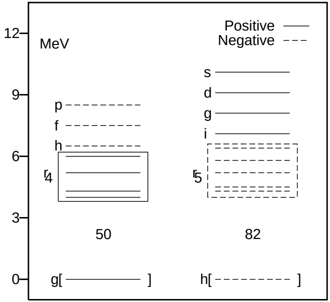
Quasi-SU3 is confirmed by an analysis of the wavefunctions: For the lowest two orbits, the overlaps between the pure quasi-SU(3) wavefunctions calculated in the restricted $\Delta j = 2$, *fp*-space and the ones in the full *pf* shell exceed 0.95 throughout the interval $0.5 < \varepsilon < 1$. More interesting still: the contributions to the quadrupole moments from these two orbits vary very little, and remain close to the values obtained at $\varepsilon = 0$ (*i.e.*, from the Quasi-SU3 Nilsson orbits).

The quasi-SU3 spherical configurations

The calculations in the restricted $(fp)^n$ spaces account remarkably well for the results in the full major shell $((pf)^n)$. The same happens in larger spaces. The intrinsic quadrupole moments Q_0 remain constant to within 5% up to a critical J value at which the bands backbend.



Heavier nuclei: Quasi+Pseudo SU(3)



Quasi-SU(3) is a variant of SU(3) that obtains for moderate spin-orbit splittings. For other forms of single particle spacings, the pseudo-SU(3) scheme will be favoured (in which case we have to use the SU3 Nilsson levels with pseudo-$p = p - 1$). The figure gives a schematic view of the single particle energies above ^{132}Sn. The space consists of two contiguous major shells—in protons (π) and neutrons (ν)—adequate for a SM description of the rare earth region.

Heavier nuclei: Quasi+Pseudo SU(3)

We can estimate the quadrupole moments for nuclei at the onset of deformation. We shall assume quasi-SU(3) operates in the upper shells, and pseudo-SU(3) in the lower ones. The number of particles in each shell for which the energy will be lowest will depend on a balance of monopole and quadrupole effects, but Nilsson diagrams suggest that when nuclei acquire stable deformation, two orbits K=1/2 and 3/2—originating in the upper shells of the figure—become occupied, *i.e.*, the upper blocks are precisely the 8-particle configurations. Their contribution to the electric quadrupole moment is then

$$Q_0 = 8[e_\pi(p_\pi - 1) + e_\nu(p_\nu - 1)],$$

with $p_\pi = 5$, $p_\nu = 6$; e_π and e_ν are the effective charges. Consider even-even nuclei with Z=60-66 and N=92-98, corresponding to 6 to 10 protons with pseudo-$p = 3$, and 6 to 10 neutrons with pseudo-$p = 4$ in the lower shells. From the SU3 Nilsson diagrams we obtain easily their contribution to Q_0,

Heavier nuclei: Quasi+Pseudo SU(3)

At fixed *n*, the value is constant in the four cases because the orbits of the triplet K=1/2, 3/2, 5/2 have zero contribution for *p*=3. The results, using effective charges of $e_\pi = 1.4$, $e_\nu = 0.6$ calculated are compared in the table with the available experimental values . The agreement is quite remarkable and no free parameters are involved. Note in particular the quality of the prediction of constancy (or rather $A^{2/3}$ dependence) at fixed *n*, which does not depend on the choice of effective charges. The discrepancy in ^{152}Nd is likely to be of experimental origin, since systematics indicate, with no exception, much larger rates for a 2^+ state at such low energy (72.6 keV).

N	Nd	Sm	Gd	Dy
92	4.47	4.51	4.55	4.58
	2.6(7)	4.36(5)	4.64(5)	4.66(5)
94	4.68	4.72	4.76	4.80
			5.02(5)	5.06(4)
96	4.90	4.95	4.99	5.03
			5.25(6)	5.28(15)
98	5.13	5.18	5.22	5.26
				5.60(5)

The region around the N=Z=40 nucleus ^{80}Zr

The pseudo+quasi SU3 valence space comprises orbits from two major shells

The relevant configurations for ^{80}Zr are:

$[pf]^{40}$ 0p-0h, doubly magic, SPHERICAL

$[pf]^{36}$ $[sdg]^{4}$ 4p-4h

$[pf]^{32}$ $[sdg]^{8}$ 8p-8h

The region around N=Z=40

The 4p-4h and 8p-8h configurations contain oblate and prolate states. The quadrupole moments can be easily computed with the help of the SU3 Nilsson diagrams. The results are:

4p-4h: Q_0=44 b^2 e fm^2 , Q_0=-44 b^2 e fm^2
8p-8h, Q_0=70 b^2 e fm^2 , Q_0=-70 b^2 e fm^2

The experimental values in the region are consistent with the 8p-8h prolate choice, but the model exhibits extreme oblate prolate coexistence, as it has been found in other nuclei in the region

Recent SMMC calculations in the pf+sdg valence space by Langanke et al.(NPA728(2003)109) also reach the same conclusion, giving an occupation number of the $1g_{9/2}$ orbit of 3.6 protons and 3.6 neutrons, fully consistent with the Pseudo+Quasi-SU3 8p-8h surmise. The Vampir calculations of Petrovici et al. (NPA710(2002)246) rather favor 12p-12h configurations

Coexistence near N=Z=20

In the 4p-4h intrinsic state of ^{36}Ar, the two neutrons and two protons in the *pf*-shell can be placed in the lowest K=1/2 quasi-SU3 level of the p=3 shell. This gives a contribution Q_0=25 b^2. In the pseudo-sd shell. p=1 we are left with four particles, that contribute with Q_0=11 b^2.
In the 4p-4h state of ^{40}Ca these values are Q_0=25 b^2 and Q_0=7 b^2, while in the 8p-8h the values are Q_0=35 b^2 and Q_0=11 b^2
Using the proper values of the oscillator length it obtains:
^{36}Ar 4p-4h band Q_0=136 e fm^2 (Q_0=173 e fm^2)
^{40}Ca 4p-4h band Q_0=125 e fm^2 (Q_0=148 e fm^2)
^{40}Ca 8p-8h band Q_0=180 e fm^2 (Q_0=226 e fm^2)
In very good accord with the data. The values in blue assume strict SU3 symmetry in both shells. The SM results almost saturate the quasi-SU3 bounds. The SU3 values are a 25% larger.

Conclusions

The geometry of the spherical mean field orbits giving rise to deformed rotors pertains to variants of Elliott's SU3 (pseudo-SU3, quasi-SU3).

In the well deformed limit, the effect of pairing is mainly to modify the moment of inertia. Neutron-proton pairing is responsible for about 50% of the total effect in N=Z nuclei.

np-nh configurations across N=Z=20 produce superdeformed shapes that can be explained in the pseudo-SU3+quasi-SU3 scheme. This scheme applies also to other mass regions, either proton rich as in ^{80}Zr (N=40, Z=40), or neutron rich as in ^{32}Mg (N=20), or ^{40}Mg and ^{42}Si (N=28).