

Chapter 2

A Pedestrian Approach to the Theory of Transfer Reactions: Application to Weakly-Bound and Unbound Exotic Nuclei

Joaquín Gómez Camacho and Antonio M. Moro

2.1 Introduction

Let us consider a simplified semiclassical time-dependent picture of transfer. Imagine a nucleus, such as ^{11}Be , that is traveling towards a target, say ^{208}Pb . When ^{11}Be is far away from the target, we can see its structure. It is made of 4 protons and 7 neutrons, that interact with each other, and are subject to Pauli principle. However, one of the neutrons is very weakly bound, so it is described by an extended halo wavefunction. The weakly bound neutron (assuming that we can identify it, because all neutrons are identical), spends a large fraction of the time well separated of the rest, although some fraction of the time it interacts strongly with the other protons and neutrons, and hence it is strongly correlated with them. In fact, due to this correlation, we suspect that the ground state of ^{11}Be , in which our nucleus presently is, is a combination of two components, in each one of them the halo neutron is coupled to a different state of the ^{10}Be core, that is how we call the other 4 protons and 6 neutrons.

Now the ^{11}Be nucleus begins to approach the ^{208}Pb target, in what is called the incident channel. The strong Coulomb force of the target begins to act, and repels the 4 protons, but does not affect in principle the 7 neutrons. However, as 6 of the neutrons are strongly correlated with the protons, the ^{10}Be core moves as a whole, and separates from the halo neutron. As the nuclei get closer, the nuclear force acts, and as a result the halo neutron excites, and also the ^{10}Be core can excite. So,

J. Gómez Camacho (✉)

Centro Nacional de Aceleradores, Universidad de Sevilla/Junta de Andalucía/CSIC,
41092 Seville, Spain
e-mail: gomez@us.es

J. Gómez Camacho · A.M. Moro

Departamento de FAMN, Universidad de Sevilla, Apartado 1065, 41080 Seville, Spain

A.M. Moro

e-mail: moro@us.es

the ^{11}Be nucleus, due to the Coulomb and nuclear interaction with the target, ends up in a combination of the initial ground state, other bound excited states and the continuum of break-up states.

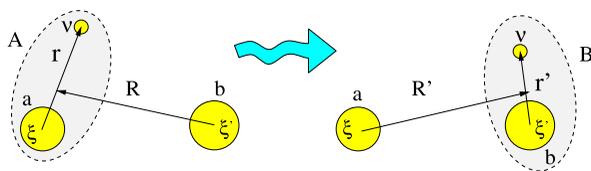
Eventually, the two nuclei are sufficiently close so that the halo neutron has the possibility of populating an unoccupied neutron state in the ^{208}Pb target. This transition is the result of many-body dynamics, involving (at least) the interaction of the neutron with the ^{10}Be core, the neutron with the ^{208}Pb target, and the ^{10}Be core with the ^{208}Pb target. Indeed, the transition will be more probable if the initial state of the halo neutron, bound to the ^{10}Be core, has a significant overlap with the final state of the neutron, in a certain bound state with the ^{208}Pb target. This overlap is not only a spatial overlap, occurring when the projectile and target are sufficiently close, but also overlap in the momentum space. The neutron will not jump easily to a state where its momentum (or its velocity) is very different from the initial one. Looking from a coordinate frame fixed in the target, the initial neutron has a momentum distribution that is centered at the projectile momentum divided by the projectile mass, with a certain spread given by the bound wavefunction in momentum space. The final wavefunction, bound to the target, is centered at zero, and has a spread given by this momentum distribution of this wavefunction. The tails of these two distributions overlap, and this favours transfer.

Once the transfer occurs, the remaining nuclei, ^{10}Be and ^{209}Pb in this case, have to fly apart. As they separate, in the so called outgoing channel, coupling effects can occur. Both ^{10}Be and ^{209}Pb can suffer excitations, produced by the Coulomb and nuclear forces. They can break-up, starting from bound states, or can bind, starting from break-up states. In addition, both in the incident and in the outgoing channels, complicated processes such as fusion can occur. These complicated processes do not need to be taken into account explicitly, but at least the loss of flux to the transfer channel has to be considered.

This semiclassical time-dependent description of a transfer reaction is what we think that is happening, but we cannot observe it directly. The only things that we can observe are the outgoing particles and, in some cases, gamma rays. So, if in our detectors we observe ^{11}Be , with the proper energy, we will conclude that we observe elastic scattering. This does not mean that ^{11}Be was in its ground state all the time. It could have excited, broken up, even transferred one neutron, but in the end it finished in the ground state.

If in our detectors we observe ^{10}Be , then we can infer that neutron transfer, or neutron break-up, has occurred. If it is neutron break-up, we expect to see a continuum of kinetic energies of ^{10}Be , centered at an energy that is 10/11 times the incident energy of ^{11}Be . However, if it is transfer, the ^{10}Be will appear with a definite energy, that is determined, for each scattering angle, by the Q-value of the reaction and kinematic factors. A well defined experiment, with sufficient energy resolution, should be able to determine individually the transfer probability, or cross section, for each scattering angle and for each excitation energy of the final nuclei ^{209}Pb and ^{10}Be . Even if there is not sufficient energy resolution to separate the ground and excited states of ^{209}Pb and ^{10}Be , the measurement in coincidence of gamma rays can allow to separate individual states. A thorough and up-to-date review of the

Fig. 2.1 Co-ordinate representation of a transfer reaction, before and after transfer occurs

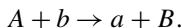


experimental procedures to measure transfer reactions is presented in the paper by W. Catford, in this volume.

A measurement of transfer reactions can give us unique information about the structure of the initial (^{11}Be) or final (^{209}Pb) composite nuclei, which are represented, respectively, by the capital letters A , B in Fig. 2.1. In particular, it can tell us which part of its wavefunction can be described in terms of the “cores” (^{10}Be and ^{208}Pb respectively), represented by the lowercase a , b in Fig. 2.1 and a neutron. Nevertheless, to obtain this structure information, one needs to deal with the complex dynamics of a quantum mechanical three-body problem, with Coulomb and nuclear forces in their full glory. To rescue the “beauty” of nuclear structure, we have to tame the “beast” of nuclear reactions. This paper is a beginners’ guide to rescue the fragile beauty of weakly bound and unbound nuclear structure, taming the formidable beast of nuclear reactions in its full three-body power.

2.2 Theoretical Formalism

Let us denote generically a transfer reaction as:



In general, this is a very complex many-body reaction. However, we can visualize it considering that, initially, we have a nucleus A , which can be described as composed of two clusters $a + v$. During the reaction, the nucleus A is fragmented into the clusters $a + v$. The cluster a survives in the final state, while the cluster v gets attached to the nucleus b to form the composite system B . For example, in the reaction $^{10}\text{Be}(d, p)^{11}\text{Be}$, b corresponds to ^{10}Be , A is the deuteron, a is the proton, B is ^{11}Be and v is the neutron.

We can describe the quantum mechanical state of the nucleus A as

$$\Phi_A = C_{av}^A \phi_a \phi_v \varphi_{av}(\mathbf{r}) + \Phi_A^C. \quad (2.1)$$

In this simplified notation, ϕ_a and ϕ_v represent the internal wavefunctions of clusters a and v , $C_{av}^A \varphi_{av}(\mathbf{r})$ represents the overlap function, which can be written in terms of a normalized relative wavefunction $\varphi_{av}(\mathbf{r})$ and a spectroscopic amplitude C_{av}^A . The product of these three terms is implicitly coupled to the angular momentum of nucleus A .

Notice that not all the state Φ_A can be described as two clusters a , v with a certain relative motion; Φ_A^C represents the part of the state that has a more complex configuration.

Similarly, the state of the nucleus B can be written as

$$\Phi_B = C_{bv}^B \phi_b \phi_v \varphi_{bv}(\mathbf{r}') + \Phi_B^C. \quad (2.2)$$

The transition matrix element that describes the transfer process will be a complex many-body matrix element, that can be written as

$$T(aB, Ab) = \langle \Phi_B \phi_a \chi_{aB} | \mathcal{T} | \Phi_A \phi_b \chi_{Ab} \rangle \quad (2.3)$$

where χ_{Ab} describes the relative motion of A and b , and similarly for χ_{aB} , and \mathcal{T} is the adequate many-body T-matrix operator. The transition matrix element, squared, is proportional to the differential transfer cross section, which is the magnitude directly measured in the experiment. The scattering formalism relating transition matrix elements and scattering observables can be seen, for example in Ref. [1].

The following approximations allow us to reduce the many-body problem to a three-body problem:

- The terms Φ_A^C and Φ_B^C , corresponding to complex configurations of A and B , do not contribute significantly to transfer.
- The normalized overlap functions $\varphi_{bv}(\mathbf{r}')$ and $\varphi_{av}(\mathbf{r})$ can be approximated by the eigenstates of two-body Hamiltonians with interactions V_{bv} and V_{av} , respectively. They will be represented by some real mean-field interactions.
- During the collision process the interactions between the clusters a , b , and v are completely described by two-body interactions V_{bv} , V_{av} and U_{ab} , that cannot alter the internal states of the clusters. In our description of transfer, we do not consider explicitly processes that lead to the excitations of the clusters b and a , so the interaction between them is represented by an effective optical potential, complex in general, that we denote by U_{ab} .

With these approximations, the transfer matrix elements can be described as:

$$T(aB, Ab) = C_{bv}^{B*} C_{av}^A T^{(3)}(aB, Ab), \quad (2.4)$$

where the three-body matrix element can be expressed in the post form

$$T^{(3)}(aB, Ab) = \langle \chi_{aB}^{(-)}(\mathbf{R}') \varphi_{bv}(\mathbf{r}') | V_{av} + U_{ab} - U_{aB} | \Psi^{(+)}(\mathbf{R}, \mathbf{r}) \rangle. \quad (2.5)$$

Here, $\Psi^{(+)}(\mathbf{R}, \mathbf{r})$ is the exact solution of the 3-body problem of a , b , v with the corresponding interactions, with boundary conditions given by a plane wave with the incident momentum in the beam direction, on the A - b co-ordinate \mathbf{R} , times the bound wavefunction $\varphi_{av}(\mathbf{r})$, plus outgoing waves in all open channels. U_{aB} is a suitable potential, arbitrary at this stage, that is used to construct the two-body relative wavefunction $\chi_{aB}^{(-)}(\mathbf{R}')$. This wavefunction has boundary conditions given

by a plane wave, with the final momentum in the direction of the detector, on the a - B co-ordinate \mathbf{R}' , plus incoming waves.

Equivalently, one can use the prior form,

$$T^{(3)}(aB, Ab) = \langle \Psi^{(-)}(\mathbf{R}', \mathbf{r}') | U_{ab} + V_{bv} - U_{Ab} | \chi_{Ab}^{(+)}(\mathbf{R}) \phi_{av}(\mathbf{r}) \rangle, \quad (2.6)$$

where the three-body and two-body wavefunctions have similar meanings as before, *mutatis mutandis*. Namely, $\Psi^{(-)}(\mathbf{R}', \mathbf{r}')$ is the exact solution of the 3-body problem of a , b , v with the corresponding interactions, with boundary conditions given by a plane wave with the final momentum in the detector direction, on the B - a relative co-ordinate \mathbf{R}' , times the bound wavefunction $\phi_{bv}(\mathbf{r}')$, plus incoming waves in all open channels. U_{Ab} is a suitable potential, arbitrary at this stage, that is used to construct the two-body relative wavefunction $\chi_{Ab}^{(+)}(\mathbf{R})$. This wavefunction has boundary conditions given by a plane wave, with the initial momentum in the direction of the beam, on the b - A co-ordinate \mathbf{R} , plus incoming waves.

It should be noticed that the previous expressions are exact, assuming a 3-body model for the transfer process. Consequently, post and prior expressions give identical results, provided that the exact three-body wavefunction ($\Psi^{(-)}(\mathbf{R}', \mathbf{r}')$ or $\Psi^{(+)}(\mathbf{R}, \mathbf{r})$) is used to evaluate the transition amplitude. In general, this equivalence will break down when these exact wavefunctions are replaced by approximated ones.

2.2.1 Distorted Wave Born Approximation DWBA

The Distorted Wave Born Approximation (DWBA) [1–5] can be obtained assuming that the three-body wavefunction can be approximated by

$$\Psi^{(+)}(\mathbf{R}, \mathbf{r}) \simeq \chi_{Ab}^{(+)}(\mathbf{R}) \phi_{av}(\mathbf{r}). \quad (2.7)$$

Thus, the transition matrix element becomes, in *post* representation,

$$\begin{aligned} T^{(3)}(aB, Ab) &\simeq T_{\text{post}}^{\text{DWBA}}(aB, Ab) \\ &= \langle \chi_{aB}^{(-)}(\mathbf{R}') \phi_{bv}(\mathbf{r}') | V_{\text{post}} | \chi_{Ab}^{(+)}(\mathbf{R}) \phi_{av}(\mathbf{r}) \rangle, \end{aligned} \quad (2.8)$$

where

$$V_{\text{post}} \equiv V_{av} + U_{ab} - U_{aB}. \quad (2.9)$$

This approximation can be considered as the leading term of an expansion of the transition amplitude in terms of V_{post} . Thus, the accuracy of the DWBA approximation depends strongly on how the auxiliary potential U_{aB} is chosen. Not only this. The choice of this potential, which was arbitrary in the exact expression of $T^{(3)}(aB, Ab)$ in the post form, becomes very important in the DWBA approximation.

An equivalent derivation can be done starting from the *prior* expression of transfer, approximating

$$\Psi^{(-)}(\mathbf{R}', \mathbf{r}') \simeq \chi_{aB}^{(-)}(\mathbf{R}') \varphi_{bv}(\mathbf{r}'). \quad (2.10)$$

Thus, the transition matrix element becomes, in *prior* representation,

$$\begin{aligned} T^{(3)}(aB, Ab) &\simeq T_{\text{prior}}^{\text{DWBA}}(aB, Ab) \\ &= \langle \chi_{aB}^{(-)}(\mathbf{R}') \varphi_{bv}(\mathbf{r}') | V_{\text{prior}} | \chi_{Ab}^{(+)}(\mathbf{R}) \varphi_{av}(\mathbf{r}) \rangle, \end{aligned} \quad (2.11)$$

where

$$V_{\text{prior}} \equiv V_{bv} + U_{ab} - U_{Ab}. \quad (2.12)$$

It can be formally demonstrated that the *prior* and *post* expressions of DWBA give exactly the same result. Hence, the choice of one of another representation in DWBA is done by computational convenience, determined by the range of the interactions. In many situations, an appropriate choice of the auxiliary potential produces a certain cancellation of the *remnant term* ($U_{ab} - U_{aB}$ or $U_{ab} - U_{Ab}$ in the post and prior representations, respectively). In those situations, the transition amplitude is mostly determined by the interaction V_{av} (post) or V_{bv} (prior) and it results numerically advantageous to choose the representation for which this interaction is of shorter range.

The accuracy of DWBA depends on the choice of the auxiliary potentials in the incident channel (U_{Ab}) and in the outgoing channel (U_{aB}). These could be, in principle, any function of the co-ordinate \mathbf{R} and \mathbf{R}' , respectively. Two approaches are usually taken:

- *The microscopic approach.* The auxiliary potential in the outgoing channel U_{aB} is taken as the expectation value, in the final bound state $\varphi_{bv}(\mathbf{r}')$, of the sum of the interactions $U_{ab} + V_{av}$. Explicitly,

$$U_{aB}(\mathbf{R}') = \int d^3 \mathbf{r}' |\varphi_{bv}(\mathbf{r}')|^2 (U_{ab} + V_{av}). \quad (2.13)$$

Similarly, U_{Ab} is taken as the expectation value, in the initial bound state, of the sum of the interactions $U_{ab} + V_{bv}$,

$$U_{Ab}(\mathbf{R}) = \int d^3 \mathbf{r} |\varphi_{av}(\mathbf{r})|^2 (U_{ab} + V_{bv}). \quad (2.14)$$

In practical applications of DWBA, it is very convenient that the auxiliary potentials are central, so that they depend on the value of the radial co-ordinate ($U_{Ab}(R)$, $U_{aB}(R')$) and not on its direction. This is achieved considering only the monopole part of the folding interaction, or, equivalently, averaging the folding potential over all the magnetic substates.

The microscopic approach has the advantage of being completely determined by the two-body interactions between the fragments. From the formal point of

view, this approach would be the natural one to follow, in order to choose U_{Ab} so that the term $U_{ab} + V_{bv} - U_{Ab}$ is minimal, for the bound state φ_{av} .

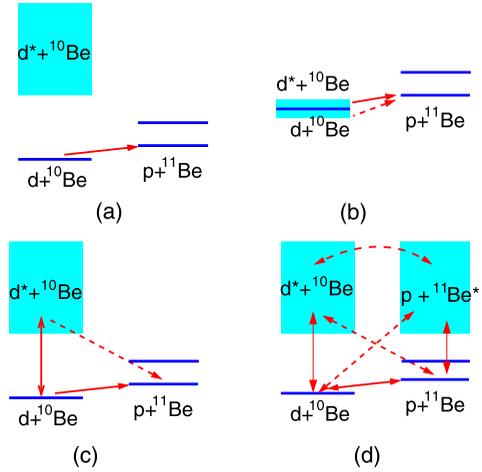
On the negative side, it is not trivial that the interaction U_{Ab} , calculated according to Eq. (2.14), reproduces accurately the elastic scattering on the Ab channel. The interactions U_{ab} , V_{av} , and V_{bv} should be taken as complex interactions, in order to reproduce elastic scattering or transfer, but in this case V_{av} and V_{bv} can not be used to obtain bound states, unless the interactions are explicitly energy dependent. Finally, this approach excludes completely any effect of break-up channels on the three-body wavefunction. Hence, this approach would be valid when the three-body scattering wavefunctions are dominated by their elastic component, either in the incident or in the exit channels.

- *The phenomenological approach.* The auxiliary potential in the incident channel U_{Ab} is obtained by fitting the elastic scattering data on the Ab channel. The auxiliary potential in the exit channel, U_{aB} , is obtained by fitting the elastic scattering on the aB channel. This approach has the advantage of allowing for a consistent description of transfer reactions, as well as of elastic scattering in the incident and outgoing channels. It takes into account, through the use of optical potentials, the effect of complex reaction processes, such as fusion, that can remove flux from the elastic and from the transfer channels. Furthermore, the effect of some three-body reactions, such as break-up, which also remove flux from elastic and transfer channels, are approximately taken into account because the optical potentials fit the experimental elastic cross sections, which are affected by all these dynamic processes. On the negative side, it is not always possible to find the elastic data for the outgoing channel. If the final state of nucleus B is not in its ground state, but on an excited state, it will not be possible to measure the corresponding elastic scattering. This is particularly true if the final state is in the continuum. Moreover, the optical potentials reproduce typically the asymptotic wavefunctions, which determines the scattering amplitudes and differential cross sections. It does not necessarily reproduce the wavefunctions in the internal radial range that is relevant for the transfer matrix elements.

The coupling scheme assumed in the DWBA method is schematically depicted in Fig. 2.2(a) for the $^{10}\text{Be}(d, p)^{11}\text{Be}$ reaction. The solid arrow indicates that only transfer from the ground state of the deuteron to the $p + ^{11}\text{Be}$ channel is explicitly included. The effect of breakup channels of the deuteron (represented by the shaded area) is completely neglected in the afore-mentioned microscopic approach, and only partially taken into account in the phenomenological approach, through its effect on the elastic wavefunction.

In general, DWBA has been, and still is, a key approach to describe transfer reactions, and it has been used extensively to extract spectroscopic information on nuclear structure, in particular spectroscopic amplitudes (see e.g. [4, 6–9]). However, this method is based on a rather crude approach to the three-body problem, and is expected to be accurate only when the elastic scattering, in the incident and outgoing channels, is dominant. For the case of exotic nuclei, which are frequently weakly bound, break-up channels can play a very important role in the three-body

Fig. 2.2 Comparison of different coupling schemes discussed in this work for the reaction $^{10}\text{Be}(d, p)^{11}\text{Be}$: (a) DWBA, (b) ADWA, (c) CDCC-BA and (d) CRC



dynamics. Hence, it is important, in order to extract reliable spectroscopic information from transfer reactions with exotic nuclei, to check the validity of the DWBA method by comparing it with other approaches that take into account the role of break-up channels.

2.2.2 *Adiabatic Distorted Wave Approximation ADWA*

The DWBA approach, as mentioned previously, relies heavily on the assumption that the elastic channel dominates the reaction. This does not only imply that the dominant cross sections is elastic, but also that, during the collision process, the three-body wavefunction can be approximated by the elastic component. Note that these two facts are not equivalent. There can be dynamic situations in which elastic cross section dominates, meaning that the asymptotic three-body wavefunction, at large distances, is dominated by the elastic component. However, this does not mean that at short projectile-target distances, which give the main contribution to the transfer matrix element, the elastic component should be dominant. Dynamic polarization effects make that the composite projectile can be strongly distorted at short distances, even when asymptotically the energy matching conditions make the elastic channel dominant.

Moreover, the phenomenological DWBA approach relies on the use of optical potentials, usually taken as local, angular momentum-independent potentials, chosen to reproduce elastic scattering. This means that the optical potentials will reasonably reproduce the phase shifts, for all partial waves, in the elastic channel. In other words, the phenomenological DWBA approach reproduces the elastic wavefunction asymptotically, at large projectile-target distances. It is not obvious that the elastic wavefunction used in the phenomenological DWBA approach reproduces correctly

the elastic component of the wavefunction, in the radial range relevant for the transfer T-matrix elements.

Due to these limitations, it would be desirable to have an alternative formulation, which maintains the relative simplicity of DWBA, and whose ingredients can be completely determined from experiment. This is achieved by the Adiabatic Distorted Wave Approximation (ADWA), which was initially formulated by Johnson and Soper [10]. This approach is formulated in principle for (d, p) , or (d, n) reactions, although it could be applied to other weakly bound composite systems. It relies on the fact that the composite projectile has a relatively low binding energy (2.22 MeV in the case of the deuteron), and so, if the collision energy is relatively high, we can expect that, during the collision process, the relative proton-neutron co-ordinate does not change significantly; it is “frozen”. Under this situation, the relevant interaction that determines accurately the projectile-target wavefunction is not the phenomenological deuteron-target interaction that would reproduce elastic scattering, but the sum of the interactions of each one of the fragments of the projectile (proton and neutron in the deuteron case) with the target.

In the *adiabatic approximation* [10] (also called *sudden approximation* by some authors) the three-body wavefunction can be written as

$$\Psi^{(+)}(\mathbf{R}, \mathbf{r}) \simeq \chi_{Ab}^{(+)}(\mathbf{R}, \mathbf{r})\varphi_{av}(\mathbf{r}), \quad (2.15)$$

where $\chi_{Ab}^{(+)}(\mathbf{R}, \mathbf{r})$ is the solution of a two-body scattering problem, on the co-ordinate \mathbf{R} , in which the interaction is given by

$$U_{Ab}(\mathbf{R}, \mathbf{r}) = U_{ab}(R_{ab}) + V_{bv}(r'). \quad (2.16)$$

Indeed, the potential that describes the scattering wavefunction, although two-body, is not central and so the calculation of the adiabatic wavefunction, for each value of the $a-v$ separation \mathbf{r} is very complicated, but it has been done [11, 12]. Besides, the adiabatic approximation to the three-body wavefunction is not accurate for large values of \mathbf{r} , where one would expect to see outgoing waves, instead of the exponential decay given by the bound two-body wavefunction $\varphi_{av}(\mathbf{r})$.

Fortunately, these shortcomings of the adiabatic wavefunctions are not important, if one is only interested in evaluating the matrix element involved in transfer. These are dominated by the $V_{av}(r)$ interaction (the proton-neutron interaction, in the deuteron case) which has a short range. Note that, even if the $a-v$ wavefunction $\varphi_{av}(\mathbf{r})$ has a relatively long range, which is the case for weakly bound halo systems, $V_{av}(r)$ has a much shorter range. Hence, for the purpose of evaluating the transfer matrix element, one can calculate the adiabatic wavefunction using the potential evaluated at $\mathbf{r} = 0$. This leads to the Johnson and Soper approximation [10], in which

$$\Psi^{(+)}(\mathbf{R}, \mathbf{r}) \simeq \chi_{Ab}^{(+)}(\mathbf{R})\varphi_{av}(\mathbf{r}), \quad (2.17)$$

where $\chi_{Ab}^{(+)}(\mathbf{R})$ is the solution of a two-body scattering problem, on the co-ordinate \mathbf{R} , in which the interaction is given by

$$U_{Ab}^{JS}(R) = U_{ab}(R) + U_{bv}(R). \quad (2.18)$$

Note that in this expression the b - v interaction V_{bv} , which would in general be energy dependent, and would be responsible for the bound state $\varphi_{bv}(\mathbf{r}')$, is replaced by the optical potential U_{bv} that describes the b - v interaction at the same incident energy per nucleon. This is justified by the adiabatic approximation; the transfer process dynamics is consistent with freezing the a - v co-ordinate, that then scatters from the b target with an interaction that is the sum of U_{bv} and U_{ab} interactions at the same energy per nucleon.

Several refinements and corrections have been performed within the ADWA formalism. For example, a finite-range version of the adiabatic potential was proposed by Johnson and Tandy [13]:

$$U_{Ab}^{JT}(\mathbf{R}) = \frac{\langle \varphi_{av}(\mathbf{r}) | V_{av}(U_{ab} + U_{bv}) | \varphi_{av}(\mathbf{r}) \rangle}{\langle \varphi_{av}(\mathbf{r}) | V_{av} | \varphi_{av}(\mathbf{r}) \rangle}. \quad (2.19)$$

However, for the purpose of the analysis of (d, p) and (p, d) reactions, the simplest Johnson-Soper expression given by Eq. (2.18) is by far the most widely used. Here, we will outline its advantages and disadvantages. On the positive side, the ADWA approach ingredients are completely determined by experiments. These ingredients are the proton-target and neutron-target optical potentials, evaluated at half of the deuteron incident energy, as well as the well known proton-neutron interaction.

The adiabatic approximation is equivalent to neglect the excitation energy of the states of the projectile [10]. The adiabatic wavefunction takes into account the excitation to breakup channels, assuming that these states are degenerate in energy with the projectile ground state, as illustrated in Fig. 2.2(b). Therefore, the ADWA approach takes into account, approximately, the effect of deuteron break-up on the transfer cross section, within the adiabatic approximation. So, it should be well suited to describe deuteron scattering at high energies, around 100 MeV per nucleon. Systematic studies [14–16] have shown that ADWA is superior to standard DWBA for (d, p) scattering at high energies.

On the negative side, the ADWA approach does not consistently describe elastic scattering and nucleon transfer. Although physically one considers that elastic scattering, transfer and break-up should be closely related, so that the increase of flux in one channel should reduce the flux in the others, this connection is not present in ADWA. On the other hand, the arguments leading to ADWA are strongly associated with the assumption that the transfer is governed by a short range operator. So, it is not obvious that the approximations remain valid for other weakly bound systems, like ^{11}Be . Even in the case of (d, p) scattering, the transfer matrix element is determined not only by the n - p interaction, but also by the proton-target and neutron-target interactions, that define the *remnant* term. It is not clear a-priori the role of these terms, that would have contributions of three-body configurations in which proton and neutron are not so close together. A promising alternative, that avoids the presence of these remnant terms, has been proposed by Timofeyuk and Johnson [17].

2.2.3 Continuum Discretized Coupled Channels Born Approximation CDCC-BA

In scattering of weakly bound nuclei, coupling to break-up channels can play an important role. DWBA may not be sufficiently accurate, as the three-body wavefunction is not dominated by the elastic channels. ADWA requires to assume the adiabatic approximation for the composite projectile, which may not be accurate if the collision energy is not sufficiently high. Besides, the simple Johnson-Soper expression requires to assume that the transfer operator is of short range, which may not be accurate beyond (d, p) reactions.

A more accurate approach for transfer is obtained if the three-body wavefunction is approximated in terms of a basis of the states of the relative motion of the $a + v$ sub-system, i.e.

$$\Psi^{(+)}(\mathbf{R}, \mathbf{r}) \approx \Psi^{(+)\text{CDCC}}(\mathbf{R}, \mathbf{r}) = \sum_{i=0}^N \chi_{Ab,i}^{(+)}(\mathbf{R}) \varphi_{av,i}(\mathbf{r}). \quad (2.20)$$

Here, the index i corresponds to a set of $a + v$ states explicitly included in a coupled channels calculation ($\varphi_{av,i}(\mathbf{r})$), which would correspond in general to a given spin and spin projection ($i = 0$ denotes the ground state of the $a + v$ system). This basis of states should include other possible bound states of the $a + v$ system, if present, as well as a suitable discrete representation of the two-body continuum states. In actual calculations, this continuum must be truncated in excitation energy and limited to a finite number of partial waves ℓ associated to the relative co-ordinate \mathbf{r} . Normalizable states representing the continuum should be obtained for each ℓ value. This can be achieved making use of a pseudo-state basis and diagonalizing the $a + v$ Hamiltonian [18]. Alternatively, continuum states of the $a + v$ Hamiltonian can be obtained, and normalizable states (bins) can be obtained by averaging these continuum states over a certain energy interval [19].

The model wavefunction given by Eq. (2.20) must verify the Schrödinger equation: $[H - E]\Psi^{(+)\text{CDCC}}(\mathbf{R}, \mathbf{r}) = 0$. To determine the radial coefficients $\chi_{Ab,i}^{(+)}(\mathbf{R})$, one multiplies this equation on the left by each of the internal wavefunctions $\varphi_{av,i}(\mathbf{r})^*$ and integrates along the coordinate \mathbf{r} . This gives rise to a set of coupled differential equations:

$$[E - \varepsilon_{av}^i - \hat{T}_{Ab} - U_{Ab}^{ii}(\mathbf{R})]\chi_{Ab,i}^{(+)}(\mathbf{R}) = \sum_{j \neq i}^N U_{Ab}^{ij}(\mathbf{R})\chi_{Ab,j}^{(+)}(\mathbf{R}), \quad (2.21)$$

where U_{Ab}^{ij} are the transition potentials defined as

$$U_{Ab}^{ij}(\mathbf{R}) = \int d\mathbf{r} \varphi_{av,i}^*(\mathbf{r})(U_{ab} + U_{bv})\varphi_{av,j}(\mathbf{r}). \quad (2.22)$$

The coupled channels solution $\chi_{Ab,i}^{(+)}(\mathbf{R})$ corresponds to the outgoing waves in all different channels i , for boundary conditions given by a plane wave in the initial

bound state $i = 0$. The potentials U_{ab} and U_{bv} are to be understood as effective interactions (complex in general) describing the elastic scattering of the corresponding $a + b$ and $b + v$ sub-systems, at the same energy per nucleon as in the incident projectile. In particular, U_{bv} will be described in general by a complex optical potential, and will differ from the interaction V_{bv} used to generate the bound state wavefunction of the $b + v$ system.

Note that, without any loss of generality, we can introduce an arbitrary auxiliary potential $U_{Ab}(R)$, so that Eq. (2.21) can be written as

$$[E - \varepsilon_{av}^i - \hat{T}_{Ab} - U_{Ab}(R)]\chi_{Ab,i}^{(+)}(\mathbf{R}) = \sum_j^N V_{\text{prior}}^{ij}(\mathbf{R})\chi_{Ab,j}^{(+)}(\mathbf{R}), \quad (2.23)$$

where $V_{\text{prior}}^{ij}(\mathbf{R})$ are the matrix elements of $V_{\text{prior}} = U_{ab} + U_{bv} - U_{Ab}$ between the states of the A system.

Once the CDCC wavefunction (2.20) is obtained, it can be inserted into Eq. (2.5) to give:

$$T^{(\text{CDCC})}(aB, Ab) = \langle \chi_{aB}^{(-)}(\mathbf{R}')\varphi_{bv}(\mathbf{r}') | V_{\text{post}} | \Psi^{+(\text{CDCC})}(\mathbf{R}, \mathbf{r}) \rangle. \quad (2.24)$$

with V_{post} given by Eq. (2.9). To clarify the link between the CDCC-BA and DWBA methods it is convenient to rewrite this expression as:

$$T^{(\text{CDCC})}(aB, Ab) = \langle \chi_{aB}^{(-)}(\mathbf{R}')\varphi_{bv}(\mathbf{r}') | V_{\text{post}} | \chi_{Ab,0}^{(+)}(\mathbf{R})\varphi_{av,0}(\mathbf{r}) \rangle + \sum_{i=1}^N \langle \chi_{aB}^{(-)}(\mathbf{R}')\varphi_{bv}(\mathbf{r}') | V_{\text{post}} | \chi_{Ab,i}^{(+)}(\mathbf{R})\varphi_{av,i}(\mathbf{r}) \rangle. \quad (2.25)$$

The first term in this expression corresponds to the *direct transfer*, proceeding directly from the ground state of the projectile (e.g. the deuteron, in a (d, p) reaction), whereas the second term accounts for the *multi-step* transfer occurring via the excited states of the projectile ($p-n$ continuum states in the case of the deuteron). These two types of processes correspond, respectively, to the solid and dashed lines in Fig. 2.2(c) for the $^{10}\text{Be}(d, p)^{11}\text{Be}$ case. Clearly, the multi-step process going through the breakup channels are omitted in the DWBA calculation. At most, the DWBA considers the effect of these channels on the elastic scattering if a suitable choice of the entrance optical potential is made. The adiabatic approximation includes in principle both mechanisms, but under the assumption that the excited (breakup) channels of the projectile are degenerate with the ground state [Fig. 2.2(b)]. The advantage of the CDCC-BA approach is that all relevant bound and continuum states in the $a + v$ system are explicitly included in the calculation.

Some early comparisons between these three methods can be found in Refs. [18, 20–22] and the main results are also summarized in Ref. [19]. Due to numerical limitations, these first studies were done using a zero-range approximation of the V_{av} potential. Overall, they find that the ADWA model describes well the direct transfer contribution. However, the multi-step contribution, which are completely absent

in DWBA, are described very inaccurately by the adiabatic approximation. At low energies ($E_d < 20$ MeV) the discrepancy between the ADWA and CDCC-BA calculation can be understood because at these energies the adiabatic approximation itself is questionable. However, even at medium energies ($E_d \approx 80$ MeV) there are situations in which transfer through breakup channels is found to be very significant, and therefore the ADWA method did not work well either. In these situations, the CDCC-BA should be better used instead. The disadvantage of the CDCC-BA calculations is that, in principle, a large basis of internal states has to be included, making this approach much more demanding numerically.

Finite-range effects have been studied within the adiabatic approximation in Refs. [23, 24] and were found to be small ($< 10\%$) at energies below 20–30 MeV/u but become more and more important as the incident energy increases. This limitation should be also taken into account in the analysis of experimental data.

2.2.4 Coupled Reaction Channels CRC

It was stated that Eqs. (2.5) and (2.6) provide the exact solution to the 3-body scattering problem, provided that $\Psi^{(+)}(\mathbf{R}, \mathbf{r})$ (in the post form) or $\Psi^{(-)}(\mathbf{R}', \mathbf{r}')$ (in the prior form) correspond to the exact three-body wavefunctions with the appropriate boundary conditions. However, in practical calculations, these exact solutions are not available and thus they need to be replaced by approximated ones, such as the factorized form used in the DWBA method, the adiabatic wavefunction or the CDCC expansion. In all these approximations, the three-body wavefunction is restricted to configurations corresponding to either the initial or the final channel. For example, in the post representation, the initial state is a solution of the three-body Schrödinger equation

$$[\hat{T} + V_{av} + U_{bv} + U_{ab} - E]\Psi^{(+)}(\mathbf{r}, \mathbf{R}) = 0, \quad (2.26)$$

where \hat{T} stands for the full kinetic energy operator. Asymptotically, the solution of this equation is of the form

$$\Psi^{(+)}(\mathbf{r}, \mathbf{R}) \rightarrow \varphi_{av}(\mathbf{r})e^{i\mathbf{K}\cdot\mathbf{R}} + \text{outgoing waves}, \quad (2.27)$$

where the “outgoing waves” contain contributions from all open channels. This includes elastic and breakup channels, but also rearrangement channels of the $a + b$ and $v + b$ pairs, if they are present. In principle, the eigenstates of the $a + v$ Hamiltonian form a complete set and hence the expansion Eq. (2.20) should contain all the relevant channels. In particular, the asymptotic part of (2.20) should contain information from all open channels, including rearrangement channels. However, rearrangement channels corresponding to the $b + v$ system would behave asymptotically as a product of the bound wavefunction $\varphi_{bv}(\mathbf{r}')$ times a plane wave in the aB co-ordinate. Although these states could be in principle expressed in the $\varphi_{av}(\mathbf{r})$ basis, this would require a very large number of energies and angular momenta [19]. In

other words, any finite CDCC approximation will describe poorly the contribution from rearrangement channels.

A heuristic way of incorporating rearrangement channels is provided by the Coupled-Reaction-Channels (CRC) framework [1, 5, 25–27]. We present a brief derivation here that highlights its connection with the other methods discussed in this work and, in particular, with the CDCC-BA method of Sect. 2.2.3. The idea of the CRC method is to use a model wavefunction which incorporates explicitly contributions from several mass partitions. For simplicity, let us assume that we wish to consider explicitly excited states (bound or unbound) of the incoming partition plus some excited states of the aB partition. Then, we may use the following *ansatz*:

$$\begin{aligned} \Psi^{(+)}(\mathbf{R}, \mathbf{r}) \approx \Psi^{(+)\text{CRC}}(\mathbf{R}, \mathbf{r}) = & \sum_i \chi_{Ab,i}^{(+)}(\mathbf{R}) \varphi_{av,i}(\mathbf{r}) \\ & + \sum_j \chi_{aB,j}^{(+)}(\mathbf{R}') \varphi_{bv,j}(\mathbf{r}'). \end{aligned} \quad (2.28)$$

This wavefunction can be interpreted as a generalization of the CDCC expansion of Eq. (2.20). The radial functions $\chi_{Ab,i}^{(+)}(\mathbf{R})$ and $\chi_{aB,j}^{(+)}(\mathbf{R}')$ are obtained by substituting the model wavefunction (2.28) into the Schrödinger equation:

$$[H - E]\Psi^{(+)\text{CRC}} = 0. \quad (2.29)$$

To get the equations satisfied by $\chi_{Ab,i}^{(+)}(\mathbf{R})$ we replace in this equation $\Psi^{(+)\text{CRC}}$ by the *ansatz* (2.28), multiply on the left by each of the functions $\varphi_{av,i}^*(\mathbf{r})$ and integrate along \mathbf{r} , giving rise to the system of equations:

$$\sum_{i'} \langle \varphi_{av,i} | H - E | \chi_{Ab,i'}^{(+)} \varphi_{av,i'} \rangle + \sum_j \langle \varphi_{av,i} | H - E | \chi_{aB,j}^{(+)} \varphi_{bv,j} \rangle = 0. \quad (2.30)$$

Now, recall that H can be written in two different forms, depending on whether one chooses the representation of the initial or final channel, namely,

$$H = \hat{T}_{Ab} + H_{av} + U_{Ab}(R) + V_{\text{prior}} \quad (\text{prior representation}) \quad (2.31a)$$

$$= \hat{T}_{aB} + H_{bv} + U_{aB}(R') + V_{\text{post}} \quad (\text{post representation}), \quad (2.31b)$$

where $H_{av} = \hat{T}_{av} + V_{av}$ and $H_{bv} = \hat{T}_{bv} + V_{bv}$ are the internal Hamiltonians of the $a + v$ and $b + v$ systems, and $U_{Ab}(R)$ and $U_{aB}(R')$ are auxiliary potentials, to be specified later. The prior and post interactions are $V_{\text{prior}} = V_{Ab} - U_{Ab}(R)$ and $V_{\text{post}} = V_{aB} - U_{aB}(R')$, with $V_{Ab} \equiv V_{bv} + U_{ab}$, $V_{aB} \equiv V_{av} + U_{ab}$.

The first term in (2.30) is a matrix element between internal states of the initial partition; hence, the natural choice for H to evaluate this part is the prior represen-

tation:

$$[E - \varepsilon_{av}^i - \hat{T}_{Ab} - U_{Ab}^i(R)]\chi_{Ab,i}^{(+)}(\mathbf{R}) = \sum_{i'} \langle \varphi_{av,i} | V_{\text{prior}} | \varphi_{av,i'} \rangle \chi_{Ab,i'}^{(+)}(\mathbf{R}) + \sum_j \langle \varphi_{av,i} | H - E | \chi_{aB,j}^{(+)} \varphi_{bv,j} \rangle, \quad (2.32)$$

where we have used the fact that $\langle \varphi_{av,i} | H_{av} | \varphi_{av,i'} \rangle = \varepsilon_{av}^i \delta_{i,i'}$ and that the kinetic energy operator \hat{T}_{Ab} does not depend on the \mathbf{r} coordinate. Note that a superscript i has been added to the auxiliary potential $U_{Ab}^i(R)$ to indicate that this potential can be taken differently for each of the equations above. For example, this potential could be taken as the monopole part of the cluster-folded potential $\langle \varphi_{av,i} | V_{Ab} | \varphi_{av,i} \rangle$.

Likewise, to get the equations for $\chi_{aB,j}^{(+)}(\mathbf{R}')$, we use the post form of the Hamiltonian in the second term of Eq. (2.30), and project the Schrödinger equation onto the functions $\varphi_{bv,j}^*(\mathbf{r}')$, giving rise to:

$$[E - \varepsilon_{bv}^j - \hat{T}_{aB} - U_{aB}^j(R')] \chi_{aB,j}^{(+)}(\mathbf{R}') = \sum_{j'} \langle \varphi_{bv,j} | V_{\text{post}} | \varphi_{av,j'} \rangle \chi_{aB,j'}^{(+)}(\mathbf{R}') + \sum_i \langle \varphi_{bv,j} | H - E | \chi_{Ab,i}^{(+)} \varphi_{bv,i} \rangle. \quad (2.33)$$

The set of equations (2.32) and (2.33) constitute the CRC equations for the three-body problem at hand. The first set of equations (2.32) correspond to the functions $\chi_{Ab,i}^{(+)}(\mathbf{R})$, which describe the relative motion between the projectile and the target for each state of the projectile i . The source term (RHS in this equation) shows that these functions are affected by two kinds of couplings. The first term, corresponds to couplings between the state i and other states of the same mass partition (i'), i.e., inelastic scattering. These coupling potentials are more explicitly given by:

$$\langle \varphi_{av,i} | V_{\text{prior}} | \varphi_{av,i'} \rangle = \int \varphi_{av,i}^*(\mathbf{r}) (V_{Ab} - U_{Ab}^i) \varphi_{av,i'}(\mathbf{r}) d\mathbf{r}. \quad (2.34)$$

The second term in the RHS of (2.32) describes the couplings between the states of the initial (Ab) partition and the second partition (aB). Explicitly,

$$\langle \varphi_{av,i} | H - E | \chi_{Ab,j}^{(+)} \varphi_{bv,j} \rangle = \int \varphi_{av,i}^*(\mathbf{r}) (H - E) \chi_{aB,j}^{(+)}(\mathbf{R}') \varphi_{bv,j}(\mathbf{r}') d\mathbf{r}. \quad (2.35)$$

We see that, in this case, we cannot extract the $\chi_{aB,i}^{(+)}(\mathbf{R}')$ function from the integral, as we did for the first term. The reason is that this function depends on the variable \mathbf{R}' , which is a function of both \mathbf{r} and \mathbf{R} . This kind of couplings are said to be non-local, because they depend on the values of $\chi_{aB,j}^{(+)}(\mathbf{R}')$ in all the configuration space, and not just in a single point \mathbf{R} . In the evaluation of this matrix element, we need to replace the Hamiltonian by either its prior or post form. Since these matrix elements are between states of two different pair Hamiltonians (H_{av} and H_{bv}) the choice is

not as clear as in the case of the local matrix elements. In either case, we will get matrix elements of the interaction potentials, but also terms involving the overlaps $\langle \varphi_{av,i} | \varphi_{bv,j} \rangle$, which give a non-zero contribution because these states are not orthogonal (they are eigenstates of different Hamiltonians). These are the so-called *non-orthogonality* terms referred in the literature in the context of the CRC formalism. A more detailed discussion of these terms can be found elsewhere [1, 27].

The same kind of couplings are present in the CRC equations for $\chi_{aB,j}^{(+)}$, Eq. (2.33). That is, transfer channels are induced by couplings with the states of the initial partition but they are also indirectly affected by the couplings with other excited states of the final partition. These two kinds of couplings are depicted in Fig. 2.2(d). In particular, we see that, in this scheme, the elastic scattering will be modified by the coupling to the inelastic channels, as in the CDCC method, but also by couplings with the rearrangement channels. In many situations, however, it is assumed that the latter are small and thus a good approximation to the first set of equations can be obtained by just neglecting these couplings altogether, i.e.

$$\begin{aligned} & [E - \varepsilon_{av}^i - \hat{T}_{Ab} - U_{Ab}^i(R)] \chi_{Ab,i}^{(+)}(\mathbf{R}) \\ & \approx \sum_{i'} \langle \varphi_{av,i} | V_{Ab} - U_{Ab}^i(R) | \varphi_{av,i'} \rangle \chi_{Ab,i'}^{(+)}(\mathbf{R}). \end{aligned} \quad (2.36)$$

Within our three-body model, the interaction V_{Ab} corresponds to the sum of the interactions between the projectile constituents (a and v) and the target b . In general, these are complicated operators, depending on the energy and angular momentum but, for the purpose of solving (2.36) they are typically approximated by some optical potentials describing the elastic scattering of each constituent by the target at the same incident energy per nucleon. That is, one makes the approximation: $V_{Ab} \approx U_{bv} + U_{ab}$. Moreover, the auxiliary potential U_{Ab}^i is taken to minimize the difference $V_{Ab} - U_{Ab}^i = U_{bv} + U_{ab} - U_{Ab}^i$, which is just the term V_{prior} defined in Eq. (2.12). For example, a possible choice would be the monopole term of the expected value,

$$U_{Ab}^i(R) = \langle \varphi_{av,i} | U_{bv} + U_{ab} | \varphi_{av,i} \rangle. \quad (2.37)$$

With this choice, the set of equations (2.36) are nothing else but the CDCC equations of Eq. (2.21). If we insert the approximated solutions $\chi_{Ab,i}^{(+)}(\mathbf{R})$ of (2.36) into (2.33) we get a first order approximation for the functions $\chi_{aB,j}^{(+)}(\mathbf{R}')$, from which the scattering amplitude for transfer can be obtained. This corresponds to the CDCC-BA approximation discussed in previous sections. It can be demonstrated (see e.g. [5]) that the scattering amplitude obtained from the asymptotics of $\chi_{aB,j}^{(+)}(\mathbf{R}')$ is entirely equivalent to the solution of the integral form given by Eq. (2.5) in which the exact wavefunction is approximated by its CDCC counterpart. Therefore, this first order solution of the CRC equations is just the CDCC-BA approximation discussed in Sect. 2.2.3.

The process could be continued iteratively, by inserting the $\chi_{aB,i}^{(+)}(\mathbf{R}')$ into the first set of equations (2.32), thus providing an improved approximation to the $\chi_{Ab,i}^{(+)}(\mathbf{R})$

functions and so on. An early comparison between the CDCC-BA and the full fledged CRC calculation can be found in [25] for the $^{16}\text{O}(d, p)^{17}\text{O}(2s, 3.27 \text{ MeV})$ at several deuteron energies. At low energies ($E_d < 40 \text{ MeV}$) the effect of the proton channel on the elastic cross section is significant but decreases rapidly as the energy increases, being negligible for $E_d > 40 \text{ MeV}$. At these energies, the CDCC-BA is found to be accurate.

2.2.5 Connection with the Faddeev Formalism

The CRC method is based on a heuristic *ansatz* for the three-body wavefunction, rather than on a rigorous treatment of the three-body scattering problem. Such a rigorous solution exists and was provided many years ago by Faddeev [28]. The idea is to express the three-body wavefunction $\Psi^{(+)}$ as a sum of three components, each of them expressed in a definite Jacobi set of coordinates $\{\mathbf{r}_i, \mathbf{R}_i\}$, with $i = 1, 2, 3$ (using our previous notation, $\mathbf{r}_1 \equiv \mathbf{r}$ and $\mathbf{R}_1 \equiv \mathbf{R}$; $\mathbf{r}_2 \equiv \mathbf{r}'$ and $\mathbf{R}_2 \equiv \mathbf{R}'$):

$$\Psi^{(+)} = \Psi^{(1)}(\mathbf{r}_1, \mathbf{R}_1) + \Psi^{(2)}(\mathbf{r}_2, \mathbf{R}_2) + \Psi^{(3)}(\mathbf{r}_3, \mathbf{R}_3), \quad (2.38)$$

verifying the triad of equations

$$[E - \hat{T} - V_{av}]\Psi^{(1)} = V_{av}(\Psi^{(2)} + \Psi^{(3)}) \quad (2.39a)$$

$$[E - \hat{T} - V_{bv}]\Psi^{(2)} = V_{bv}(\Psi^{(1)} + \Psi^{(3)}) \quad (2.39b)$$

$$[E - \hat{T} - V_{ab}]\Psi^{(3)} = V_{ab}(\Psi^{(1)} + \Psi^{(2)}). \quad (2.39c)$$

Note that adding the three equations one recovers the original Schrödinger equation (2.26). The advantage of this decomposition is that each equation contains only one pair interaction and, therefore, the asymptotic form of the associate Faddeev component can only contain bound states supported by that interaction. Thus, $\Psi^{(1)}$ contains the v - a short range correlations, in particular, the v - a bound states, $\Psi^{(2)}$ contains the v - b short range correlations, including v - b bound states, and $\Psi^{(3)}$ contains the a - b short range correlations, and would include the a - b bound states, assuming that they were relevant for the reaction.

At first sight, the link between this set of equations and the methods discussed in the preceding sections, is not obvious. This connection became more clear after the work of Austern, Kawai and Yahiro [29, 30]. They use the alternative but equivalent set of Faddeev equations

$$[E - \hat{T} - V_{av} - \mathcal{P}(V_{bv} + V_{ab})\mathcal{P}]\tilde{\Psi}^{(1)} = V_{av}(\tilde{\Psi}^{(2)} + \tilde{\Psi}^{(3)}) \quad (2.40a)$$

$$[E - \hat{T} - V_{bv}]\tilde{\Psi}^{(2)} = (V_{bv} - \mathcal{P}V_{bv}\mathcal{P})\tilde{\Psi}^{(1)} + V_{bv}\tilde{\Psi}^{(3)} \quad (2.40b)$$

$$[E - \hat{T} - V_{ab}]\tilde{\Psi}^{(3)} = (V_{ab} - \mathcal{P}V_{ab}\mathcal{P})\tilde{\Psi}^{(1)} + V_{ab}\tilde{\Psi}^{(2)} \quad (2.40c)$$

where \mathcal{P} is a projector defined as

$$\mathcal{P} = \sum_{i=1}^N |\varphi_{av,i}\rangle \langle \varphi_{av,i}|, \quad (2.41)$$

with N denoting a finite number of states of the $a + v$ system. In the present context, this projector corresponds to the model-space spanned by the CDCC wavefunction. The properties of this projector are discussed in [29]; it selects low angular momenta ℓ associated with the $a-v$ relative coordinate. Note that the individual components $\{\tilde{\Psi}^{(i)}\}$ will differ from the original ones $\{\Psi^{(i)}\}$ but the total wavefunction, obtained as the sum of the three components, will be the same. We can identify $\mathcal{P}V_{bv}\mathcal{P} \simeq U_{bv}$ and $\mathcal{P}V_{ab}\mathcal{P} \simeq U_{ab}$ with the effective complex interactions that would describe on-shell elastic matrix elements when the $v-b$ and $a-b$ energies per nucleon are close to the projectile incident energy per nucleon. This identification is possible because we are projecting on states in which the $v-a$ relative energy and angular momentum is not large.

Addition of (2.40b) and (2.40c) gives:

$$[E - \hat{T} - V_{av} - U_{bv} - U_{ab}]\tilde{\Psi}^{(1)} = V_{av}\tilde{\Psi}^{(2+3)} \quad (2.42a)$$

$$[E - \hat{T} - V_{bv} - V_{ab}]\tilde{\Psi}^{(2+3)} = [V_{bv} + V_{ab} - \mathcal{P}(V_{bv} + V_{ab})\mathcal{P}]\tilde{\Psi}^{(1)} \quad (2.42b)$$

with $\Psi^{(2+3)} \equiv \Psi^{(2)} + \Psi^{(3)}$. As the model-space is augmented, the term $[V_{bv} + U_{ab} - \mathcal{P}(V_{bv} + V_{ab})\mathcal{P}]$ appearing in the RHS of (2.42b) becomes smaller and smaller, thus suppressing the component $\Psi^{(2+3)}$. Under these circumstances, the RHS in the first equation can be neglected giving the zero-th order approximation

$$[E - \hat{T} - V_{av} - U_{bv} - U_{ab}]\tilde{\Psi}^{(1)} \approx 0. \quad (2.43)$$

This corresponds to the CDCC approximation. It becomes clear that the accuracy of the CDCC-BA approximation depends on the ability of the CDCC expansion to represent the full solution, at least within the region of $\{\mathbf{r}, \mathbf{R}\}$ for which the transition operator is important. This is expected to happen when the short range correlations between $b-v$ and $a-b$ are not important. In particular, the pair interactions U_{bv} and U_{ab} should not support any bound states. In fact, it was argued by Austern and collaborators [29] that the apparent success of the CDCC method to describe elastic and breakup reactions is largely due to the use of complex optical potentials to represent the U_{bv} and U_{ab} interactions.

Recently, it has become possible to solve the Faddeev equations for a number of nuclear reactions [31–34], thus providing a very useful assessment for more approximate methods. In Ref. [35], a systematic comparison was performed for (d, p) reactions on ^{10}Be , ^{12}C and ^{48}Ca targets, at several incident energies, and it was found that CDCC-BA and Faddeev agree very well at small incident energies (a few MeV per nucleon) but the agreement progressively deteriorates with increasing incident energies. Moreover, as the energy increases, the absolute cross section drops down due to the less favorable energy/momentum matching conditions [19]. So, at these

energies, small discrepancies in the absolute cross section imply a sizable relative uncertainty. This suggests that transfer reactions at high energies ($E > 100$ MeV) are not suitable for the extraction of structure information, such as spectroscopic factors. In Ref. [31] the two methods were compared for the $^{11}\text{Be}(p, d)^{10}\text{Be}$ reaction at 38.4 MeV/u and it was found that the former underestimates the Faddeev result by about 30 %. This relatively large discrepancy could be partially due to the absence of an imaginary part in one of the fragment-target interactions (p - n), which favours the coupling with rearrangement channels and makes less justified the approximation (2.43), as pointed out in Ref. [29]. Further calculations and benchmarks comparisons, for other systems and energies, are clearly needed to establish the limits of validity of the CDCC-BA and more approximated methods (DWBA, ADWA).

The Faddeev method can be also linked to the CRC formalism. For this purpose, we introduce a projector extended to the bound states, and possibly the narrow resonances, of the $a + b$ system

$$\mathcal{Q} = \sum_{i=1}^N |\varphi_{ab,i}\rangle \langle \varphi_{ab,i}|, \quad (2.44)$$

and the complementary projector $\mathcal{P} = 1 - \mathcal{Q}$. Then, we can identify the projected interaction $U_{ab} = \mathcal{P}V_{ab}\mathcal{P}$, which can eventually be approximated by a complex effective interaction that does not support any bound state or narrow resonance. Making use of these projectors, we rewrite the Faddeev equations as:

$$[E - \hat{T} - V_{av}]\tilde{\Psi}^{(1)} = (V_{av} + U_{ab})\tilde{\Psi}^{(2)} + V_{av}\tilde{\Psi}^{(3)} \quad (2.45a)$$

$$[E - \hat{T} - V_{bv}]\tilde{\Psi}^{(2)} = (V_{bv} + U_{ab})\tilde{\Psi}^{(1)} + V_{bv}\tilde{\Psi}^{(3)} \quad (2.45b)$$

$$[E - \hat{T} - V_{ab}]\tilde{\Psi}^{(3)} = (V_{ab} - U_{ab})\tilde{\Psi}^{(1)} + (V_{ab} - U_{ab})\tilde{\Psi}^{(2)}. \quad (2.45c)$$

Again, one can see that, adding the three equations, one recovers the original form of the Faddeev equations. When we are interested in elastic scattering, inelastic scattering or transfer reactions, the wavefunction components $\tilde{\Psi}^{(1,2)}$ will be important, as they appear either in the incident or the outgoing channels. However, $\tilde{\Psi}^{(3)}$ only contributes through the couplings. Thus, insofar as the term $V_{ab} - U_{ab}$ is small, this component can be neglected. Note that this will be accurate provided that $a + b$ bound states and narrow resonances do not play a role in the reaction. Neglecting $\tilde{\Psi}^{(3)}$, and including arbitrary potential matrices \mathbf{U}_{Ab} and \mathbf{U}_{aB} in both sides of Eqs. (2.45a)–(2.45c), we get

$$[E - \hat{T} - V_{av} - \mathbf{U}_{Ab}]\tilde{\Psi}^{(1)} = (V_{av} + U_{ab} - \mathbf{U}_{aB})\tilde{\Psi}^{(2)} \quad (2.46a)$$

$$[E - \hat{T} - V_{bv} - \mathbf{U}_{aB}]\tilde{\Psi}^{(2)} = (V_{bv} + U_{ab} - \mathbf{U}_{Ab})\tilde{\Psi}^{(1)}. \quad (2.46b)$$

Note that if we select the potential matrices \mathbf{U}_{aB} and \mathbf{U}_{Ab} as those whose matrix elements are precisely the transition potentials in the CDCC formalism, we obtain a

set of equations that look very similar to the CRC equations. A detailed comparison of Faddeev and CRC is in progress, and will be published by the present authors elsewhere.

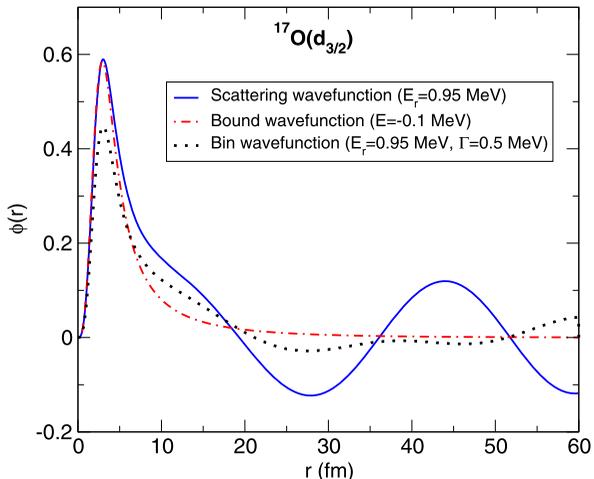
2.3 Transfer to Unbound States

So far, we have considered transfer reactions as a tool to investigate bound states of a given nucleus. However, in a rearrangement process the transferred particle can populate also unbound states of the final nucleus. This opens the possibility of studying and characterizing structures in the continuum, such as resonances or virtual states.

As in the case of transfer to bound states, the simplest formalism to analyze these processes is the DWBA method. In this case, the bound wavefunction $\varphi_{bv}(\mathbf{r}')$ appearing in the final state in Eqs. (2.8) or (2.10) should be replaced by a positive-energy wavefunction describing the state of the transferred particle v with respect to the core b . In principle, for this purpose one could use the suitable scattering state of the $v + b$ system at the appropriate relative energy. However, this procedure tends to give numerical difficulties in the evaluation of the transfer amplitude due to the oscillatory behaviour of both the final distorted wave and the wavefunction $\varphi_{bv}(\mathbf{r}')$. To circumvent this problem, several alternative methods have been used. We enumerate here some of them:

- (i) The bound state approximation [36]. In the case of transfer to a resonant state, this method replaces the scattering state $\varphi_{bv}(\mathbf{r}')$ by a weakly bound wavefunction with the same quantum numbers ℓ and j . In practice, this can be achieved by starting with the potential that generates a resonance at the desired energy and increase progressively the depth of the central potential until the state becomes bound.
- (ii) Huby and Mines [37] use a scattering state for $\varphi_{bv}(\mathbf{r}')$, but it is multiplied by a convergence factor $e^{-\alpha r'}$ (with α a positive real number), which artificially eliminates its contribution to the integral coming from large r' values, and then extrapolate numerically to the limit $\alpha \rightarrow 0$.
- (iii) Vincent and Fortune [38] questioned the bound state approximation arguing that, in general, the bound state and resonant form factors can be very different and, even in those cases in which the fictitious form factor gives the correct shape, they can lead to very different absolute cross sections. They suggest using the actual scattering state, but choosing an integration contour along the complex plane in such a way that the oscillatory integrand is transformed into an exponential decay, thus improving the convergence and numerical stability of the calculation.
- (iv) In a real transfer experiment leading to positive-energy states, one does not have access to a definite final energy, but to a certain region of the continuum. That is to say, the extracted observables, such as energy differential cross sec-

Fig. 2.3 Radial part of the $d_{3/2}$ single-particle resonance wavefunction in ^{17}O at $E_r = 0.95$ MeV compared with a slightly bound wavefunction ($E = -0.1$ MeV) and a bin wavefunction, centered at the nominal energy of the resonance and with a width of 0.5 MeV

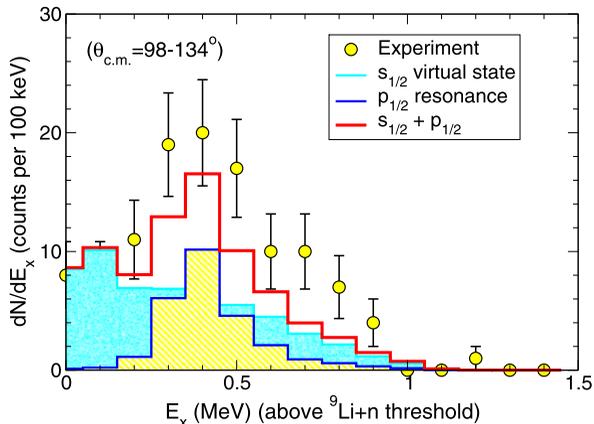


tions, are integrated over some energy range which, at least, is of the order of the energy resolution of the experiment. This suggests a method of dealing with the unbound states consisting of discretizing the continuum states in energy bins, as in the CDCC approximation.

In Fig. 2.3, we show as an example the radial part of a $3/2^+$ resonance in ^{17}O , described in terms of a $d_{3/2}$ neutron coupled to a zero-spin ^{16}O core. The solid line is a scattering wavefunction evaluated at the nominal energy of the resonance ($E_{rel} = 0.95$ MeV). Note the oscillatory behaviour at large distances. The dotted line is a bin wavefunction, constructed by a superposition of scattering states, within the range of 0.5 MeV around the resonance energy. It is seen that, asymptotically, the oscillations are damped with respect to the original scattering states. Finally, the dot-dashed line is a bound state wavefunction, with a $1d_{3/2}$ single-particle configuration, and a separation energy of 0.1 MeV. This wavefunction is very similar to the scattering state at short distances, but decays exponentially at large distances.

An advantage of the method (iv) is that it can be equally applied to both resonant and non-resonant continuum final states. An example is shown in Fig. 2.4, which corresponds to the differential cross section, as a function of the n - ^9Li relative energy, for the reaction $^2\text{H}(^9\text{Li}, p)^{10}\text{Li}^*$ at 2.36 MeV/u measured at REX-ISOLDE [39]. The lines are the results of CDCC-BA calculations, including the transfer to $^{10}\text{Li}^*$ continuum states, showing the separate contribution of the s-wave ($\ell = 0$) continuum and p-wave ($\ell = 1$) continuum. The strength of the measured cross section close to zero energy is due to the presence of a virtual state in the $^{10}\text{Li}^*$ continuum, whereas the peak around 0.4 MeV is due to a $p_{1/2}$ resonance. This is an example of how the use of transfer reactions can provide information of the continuum structure of weakly-bound or even unbound systems.

Fig. 2.4 Illustration of the transfer-to-the-continuum method, using a *binning* discretization, for the reaction ${}^2\text{H}({}^9\text{Li}, p){}^{10}\text{Li}^*$



2.3.1 Recent Applications to Weakly Bound Halo Nuclei

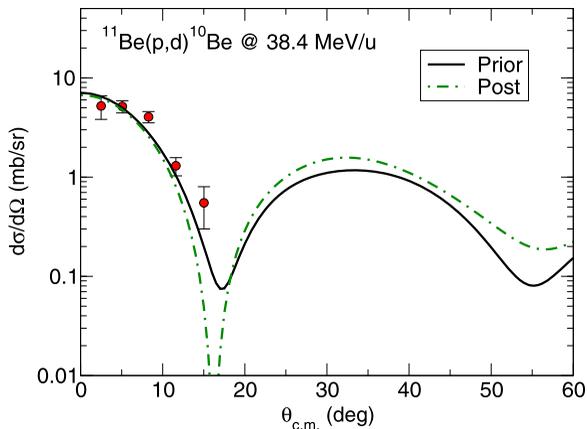
Many current nuclear reactions studies are done using exotic nuclei. In the light region of the nuclear chart, many of these exotic systems are weakly bound. As in the deuteron case, transfer reactions involving these nuclei must incorporate in some way the effect of the coupling to the unbound states of the weakly bound nucleus. This can be done using the ADWA or CDCC approximations discussed in the previous sections, that is, replacing the exact three-body wavefunction appearing in the transition amplitude by their adiabatic or CDCC counterparts. Although one expects that the CDCC-BA method provides more accurate results, most of these analyses have been done using the DWBA or ADWA methods, partially due to the computational difficulties inherent to the CDCC-BA method. In fact, early applications of the CDCC-BA method used invariably the zero-range approximation for the transition operator. However, there are nowadays computer codes, that permit these kind of calculations [40] using finite range transfer. An example of this kind of applications is given by the ${}^{14}\text{N}({}^7\text{Be}, {}^8\text{B}){}^{13}\text{C}$ reaction, which involves the weakly bound halo nucleus ${}^8\text{B}$. This reaction was measured at Texas A&M at an energy of 84 MeV with the purpose of extracting the spectroscopic factor and the so-called astrophysical S-factor for the ${}^8\text{B}$ nucleus. The reaction was later studied in [41, 42] using the CDCC-BA approximation in prior form.

An additional complication arises when these reactions involve deuterons in either the initial or final state, as for example in the case of (p, d) or (d, p) reactions. In this case both, the deuteron and halo nucleus continua, could play a role in the reaction dynamics. Examples of these reactions are ${}^7\text{Be}(d, n){}^8\text{B}$ [43], ${}^{10}\text{Be}(d, p){}^{11}\text{Be}$ [44] and ${}^{11}\text{Be}(p, d){}^{10}\text{Be}$ [45].

To deal with these reactions one possibility is to use the ADWA and CDCC-BA formalisms discussed in previous sections. For concreteness, let us consider the ${}^{11}\text{Be}(p, d){}^{10}\text{Be}$ case. In post form, the CDCC-BA transition amplitude reads

$$T^{(\text{post})}(p \rightarrow d) \simeq \langle \chi_{da}^{(-)}(\mathbf{R}') \varphi_{pn}(\mathbf{r}') | U_{pa} + V_{na} - U_{da} | \psi_{na}^{\text{CDCC}}(\mathbf{R}, \mathbf{r}) \rangle, \quad (2.47)$$

Fig. 2.5 CDCC-BA calculations for the $^{11}\text{Be}(p, d)^{10}\text{Be}$ transfer reaction at 38.4 MeV per nucleon. Experimental data are from Refs. [45, 48]



where in this case $a \equiv {}^{10}\text{Be}$. Alternatively, one can use the prior form, namely,

$$T^{(\text{prior})}(p \rightarrow d) \simeq \langle \Psi_{pn}^{\text{CDCC}}(\mathbf{R}', \mathbf{r}') | U_{pa} + V_{np} - U_{pA} | \chi_{Ap}^{(+)}(\mathbf{R}) \varphi_{av}(\mathbf{r}) \rangle \quad (2.48)$$

with $A \equiv {}^{11}\text{Be}$.

Note that, in the post form, the ^{11}Be continuum is explicitly taken into account, whereas in the prior representation the deuteron continuum is considered explicitly.¹ It is not obvious to decide beforehand which of these approximations is more suitable for actual calculations. In Ref. [47], both amplitudes were compared for this reaction at a proton energy of 38.4 MeV. The angular distributions obtained from these calculations are shown in Fig. 2.5, and compared with the data from Refs. [45, 48]. As it can be seen, both representations yield very similar results. However, the convergence was found to be much faster in the prior representation, a result that can be ascribed to the shorter range of the transition operator in that case.

Another formalism specifically designed to describe (d, p) and (p, d) transfer reactions with halo nuclei is the method proposed by Timofeyuk and Johnson [17]. This approach is based on an alternative exact representation of the transfer amplitude in which the transition operator is the p - n potential and any effects due to remnant terms are included in the wave function for the initial or final channel. This is achieved by choosing the auxiliary potential appearing in the transition operator to cancel out exactly the remnant term. So, for instance, in the previous example, in the prior representation one would make the choice $U_{pA} = V_{pa}$. With this choice the transfer operator becomes simply V_{pn} . However, the initial wave function is no longer given by the factorized form $\chi_{Ab}^{(+)}(\mathbf{R}) \varphi_{av}(\mathbf{r})$, but becomes a

¹Strictly speaking, both two-body continua are part of the same three-body continuum, namely, $p + n + {}^{10}\text{Be}$. In principle, a complete basis of either sub-system would be sufficient to describe the three-body continuum. In practice, an accurate description of the full three-body continuum might require a very large basis and so, in actual calculations, using a truncated basis, a suitable choice of the continuum representation can be important [46].

complicated three-body wavefunction. To evaluate the resulting amplitude one may use the CDCC expansions of the initial and final states [47] or the much simpler adiabatic wave functions [17].

A similar approach was followed in Ref. [43] to study the reaction ${}^7\text{Be}(d, n){}^8\text{B}$. In this case, the initial three-body wavefunction was approximated by the CDCC expansion in $p + n$ states, whereas the ${}^8\text{B}$ continuum was treated in the simple adiabatic prescription of Johnson and Soper [10].

2.4 Summary and Conclusions

We have presented the current status of the theoretical description of transfer reactions. We have shown, that, under certain assumptions, the complex many-body problem corresponding to a rearrangement collision can be approximated by a three-body problem in which a valence particle is transferred from one core to another. The connection between the many-body and the three-body problem is done through the introduction of spectroscopic amplitudes. The experimental determination of these spectroscopic amplitudes from the observed cross section is the key objective of the transfer reaction formalism.

The solution of the three-body problem and, in particular, the evaluation of the T-matrix element from a bound state of the valence particle with a core (initial partition), to a bound state of the valence particle with the other core (final partition), has been discussed with an increasing degree of complexity. We start from an exact formal expression, that gives the transfer transition amplitude as a matrix element of the interaction with the exact three-body wavefunction with boundary conditions on the initial partition on one side, and a two-body distorted wavefunction with boundary conditions on the final partition on the other side. In the DWBA method, the exact three-body wavefunction is approximated by a two-body distorted wavefunction, multiplied by a bound state wavefunction. This approximation, which has been the workhorse for transfer reactions in nuclear physics for years, is not accurate for weakly bound exotic nuclei, since it does not take into account the effects of break-up on the three-body wavefunction.

An improvement over the DWBA method is provided by the ADWA approximation, which relies on the adiabatic approximation of the three-body wavefunction, and assumes that the transfer interaction has a short range. This gives rise to an approximate three-body wavefunction that is accurate in the region where the valence particle and the core are very close. This leads to a formal expression of the transfer three-matrix, which has the same complexity as the DWBA, but with distorting potentials not related to the elastic scattering. The accuracy of ADWA depends on the validity of the adiabatic approximation, and the short range of the interaction. In general, ADWA is well suited for deuteron scattering. However, for high energies (about 100 MeV per nucleon), there is a significant contribution of transfer through the continuum, where excitation energy is high enough so that the adiabatic approximation may not be accurate.

The CDCC-BA approximation describes the three-body wavefunction using a finite representation for the continuum states in the initial or final partition. CDCC-BA has a wider validity compared to ADWA, as it does not rely on the adiabatic approximation, and does not require short range transition operators. However, it is much more demanding computationally and, as one goes to higher scattering energies, it would require to introduce explicitly high energy continuum states. It will, in principle, describe accurately the contribution of break-up states to transfer, provided convergence is found on the various parameters involved in continuum discretization. However, the CDCC-BA scheme describes only the three-body wavefunction on one partition, neglecting possible effects of multi-step processes involving transfer back and forth between different partitions. The CDCC-BA method should give an accurate description of transfer, including break-up effects, at energies from 10 MeV/u onwards.

The CRC approximation describes the 3-body wavefunction as a superposition of wavefunctions corresponding to different mass partitions. The solution of CRC equations involves the consideration of non-local potentials and non-orthogonality terms, and has to be carried out iteratively. CRC has proven very useful to describe elastic scattering and fusion in the presence of transfer cross sections that are comparable to the elastic cross section. This will be the case when the scattering energy is small, of a few MeV/u, and matching conditions favour transfer to bound states.

The correct formal solution of the three-body problem was formulated by Faddeev, in terms of a system of equations that involve 3 components of the wavefunction expressed in terms of the 3 possible sets of Jacobi coordinates. Each component of the wavefunction contains the short range two-body correlations for a given pair of particles, in particular the bound states. We have shown that the CRC equations can be obtained from the Faddeev equations by neglecting the component of the wavefunction containing core-core correlations. The CDCC-BA equations are obtained ignoring, for the 3-body wavefunctions, the correlations in two partitions, so that only one component of the Faddeev equations is considered. We justify the use of complex effective interactions, as the result of projecting two-body interaction in the restricted model space.

We have discussed some applications of the CDCC-BA approach to the description of transfer involving weakly bound systems, and also to unbound states. We have shown that CDCC-BA is able to include consistently the contribution of resonant and non-resonant continuum. Application of CDCC-BA allows to obtain, for example, the energy and width of a resonance, from the measurement of cross sections of transfer to the continuum as a function of the energy of the ejectile.

From our experience, we consider that the optimal energy range to measure transfer reaction of exotic nuclei would be a few tens of MeV per nucleon. At higher energies (100 MeV per nucleon), transfer cross sections diminish, and break-up is dominant. At lower energies (few MeV per nucleon), multi-step processes become important, compound nucleus effects become relevant, and the relation of cross sections with spectroscopic factors becomes more obscure.

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